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FINAL REPORT

for

**SHUTTLE ROCKET BOOSTER COMPUTATIONAL FLUID DYNAMICS
NAG8-629**

by

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ABSTRACT

This is the final report for the shuttle rocket booster computational fluid dynamics. The basic formulations and preliminary results were submitted in November, 1987. Additional results and a revised and improved computer program listing are reported herein. Numerical calculations for the flame zone of solid propellants are carried out using the Galerkin finite elements, with perturbations expanded to the zeroth, first, and second orders. The results indicate that amplification of oscillatory motions does indeed prevail in high frequency regions. For the second order system, the trend is similar to the first order system for low frequencies, but instabilities may appear at frequencies lower than those of the first order system. The most significant effect of the second order system is that the admittance is extremely oscillatory between moderately high frequency ranges.

CHAPTER I

INTRODUCTION

Determination of oscillatory pressure and velocity variations from their mean values in solid propellant combustion chambers has been the subject of study for many years. Because of computational difficulties, however, rigorous and accurate methods of solution still remain in various stages of development. Often details of physics are obscured in complicated computational strategies. In view of the fact that reliable experimental measurements are difficult to obtain, it is crucial to possess an analytical tool to accurately predict combustion dynamics and combustion efficiency in designing successful solid propellant rocket motors.

Earlier works on combustion dynamics include Hart and McClure [1], Denison and Baum [2], Culick [3] and T'ien [4] among others. These studies are centered around one-dimensional, linear oscillatory burning. Culick [5,6], Baum and Levine [7], and Yang et al. [8] subsequently studied the nonlinear growth and limiting amplitude of acoustic waves in a combustion chamber. Nonlinear behavior as characterized by higher order perturbation expansions of governing equations has been investigated by Flandro [9] and Chung and Kim [10] in recent years. They assumed that the pressure varies sinusoidally with time, the pressure

amplitudes being a function of position.

In the present approach, the pressure is assumed to vary in the unstable state. This allows the presence of a long flame such as in double-base solid propellants, in which high pressures and high frequencies may be accommodated. Galerkin finite elements [11] are utilized to model numerically the geometries of burning surfaces and the flame thickness. The nonsteady governing equations are linearized by means of the zeroth, first, and second order perturbation expansions. The boundary conditions, including the burning surfaces and flame edges, are imposed by means of the Lagrange multipliers.

In the simple example problems demonstrated in this paper, the gaseous flame is assumed to follow the Arrhenius law with one-step forward chemical reactions. No condensed phase reaction is included in the formulation. The calculated results confirm the prediction reported by other investigators in the literature for the low frequency region. Extended studies are then carried out for the high frequency region. It has been found that amplification of oscillatory motions does prevail in the high frequency region.

The present study does not include turbulent boundary layers which may play an important role in erosive burning and combustion instability; this is the subject of future study.

CHAPTER II

ANALYSIS AND DISCUSSION OF RESULTS

To demonstrate the validity of the theory presented above, a two-dimensional rectangular geometry, shown in Fig. 1, is analyzed, in which a burning surface and flame edge can be established as boundaries. This configuration was chosen to resemble a one-dimensional behavior so that the results may be compared directly with those reported in the literature [4,9]. The constants and parameters used in this analysis are shown in Table 1.

The computations begin with finding the natural frequencies of the system. The resonance frequencies of the system can be obtained by examining the acoustic admittance or response function at each natural frequency. As mentioned earlier, the natural frequencies of the system are given by the homogeneous solution of the total matrix equation, neglecting the boundary conditions. The frequency range of interest in this computation is shown to be between $\omega = 10^{-3}$ and $\omega = 10^2$, corresponding to approximately 80 Hz and 8 MHz, respectively. Thus, sixteen different frequencies in this range are chosen to evaluate the present study, and these are $\omega = 10^{-4}$, 5×10^{-4} , 10^{-3} , 5×10^{-3} , 10^{-2} , 5×10^{-2} , 10^{-1} , 5×10^{-1} , 1, 5, 10, 20, 30, 40, 50, and 100.

Figure 2 shows a typical steady-state distribution of

the field variables and the reaction rate along the flame thickness. In general, these data may be obtained by solving the energy equation together with the reaction rate. Ideally, a fourth order Runge-Kutta scheme may be used for this purpose. The result will be used as the basis of further calculations for combustion instability in the unsteady-state.

Distributions of the field variables versus frequency for the first order system are shown in Figs. 3-8. These results are obtained by imposing an acoustic pressure amplitude of unity at the flame edge as the Dirichlet condition. Conventionally, the thickness of the burning zone has been assumed to be negligibly small compared with the wavelength of the acoustic oscillation in the intermediate frequency range. Thus, the uniform pressure is approximated throughout the domain of study and is assumed to vary with time only. On the contrary, since the oscillatory pressure is regarded as a spatially nonhomogeneous time-dependent source in the present study, it is possible to investigate the response of a specific solid propellant at significantly high frequencies and to find the response even in the long flame of a double-base propellant. Figure 3 shows variations of the pressure distribution versus the acoustic frequencies. It is seen that the amplitude remains constant for $\omega < 10$. Random variations of these amplitudes occur for higher frequencies. Here, the imaginary parts representing the

phase shift are zero.

Figure 4 demonstrates distributions of various component fluctuations of the first order system at $\omega = 1$. A pressure wave with a certain amplitude striking the solid surface will give rise to a response in the burning rate. Generally, the response of the burning rate is nonlinear and has a complete Fourier series form that may not be expressed by a single frequency component. In Fig. 4, however, the pressure is uniform in the flame zone, although significant variations may occur at higher frequencies.

The temperature amplification at the surface changes the burning rate, while the velocity at the flame edge represents the acoustic admittance for pressure coupling, whose magnitude and sign indicate the instability of the system. Since the pressure remains constant, implying that the acoustic wavelength is larger than the flame thickness in this case, the imaginary part of the velocity approaches a constant slope at the edge by the assumption of an isentropic condition at the flame edge. It is interesting to note that the trend of variations of temperature and velocity appears to be similar, which is inversely proportional to the variations of species and density as expected. These results are comparable to those of T'ien [4] and Flandro [9].

Density distributions for various frequencies are given in Fig. 5. At $\omega < 0.5$, changes of the distributions

versus frequency are negligible; however, near $\omega = 1$ a significant reduction of the amplitude occurs, and then for $\omega > 1$ the amplitudes increase moderately. The distributions in the lower frequency region are very similar to those in the steady-state, from which the quasi-steady assumption may be deduced. Note that the reverse peak moves in the flame zone, which implies a change of the reaction zone. Close to the surface, the effect of preheat in the solid phase increases, resulting in a higher burning rate. Positive amplitudes at the interface in each frequency are caused by the rise in pressure and/or the increase of the burning rate.

It is interesting to see that the amplitudes decrease while the frequency increases up to the unity and then increase along the frequency, from which the flame is expected to react sensitively at special frequencies. The negative amplitude is shown most notably at $\omega = 1$. This may result from the net blowing effect, and the velocity at that point should be positive (see Fig. 8).

The normal velocity distributions are shown in Fig. 8. The profile may be classified into positive and negative slopes, the only exception being at $\omega = 1$. The latter contains most of the distributions in the lower frequency region. Negative amplitudes appear mostly at the interface, relating to an increase of the density. Although the pressure remains constant in lower frequencies, the velocity varies severely. At $\omega > 20$, the

variation is more significant since from that frequency the pressure varies in the flame zone. At $\omega = 0.01$, note that the slope is positive even though it is a lower frequency. This result implies instability of the system for the quasi-steady case. At $\omega = 1$, a positive amplitude near the interface results from adjusting between the density increase and the higher gasification rate. As indicated in Fig. 4, the slopes of the imaginary parts near the flame edge are constant.

Rearranging the real part of the velocity at the flame edge gives the distribution of the acoustic admittance, whose magnitude and sign indicate the amplifications or damping ability of the flame subject to the acoustic disturbance. Figure 9 shows the curve-smoothed acoustic admittance and burning rate at each frequency.

At $\omega = 1$, there is a definite increase in the burning rate. The burning rate increases by increasing the heat transfer effect due to closer movement of the reaction zone to the surface. There are actually several secondary effects such as structural change in the solid phase and change of the chemical kinetics in the flame. These effects cause the feedback to the solid phase, resulting in a change of the burning rate. These effects, however, are not considered.

Figure 9 also reveals a resonance in the condensed phase near $\omega = 0.01$, indicating that the system is unstable. This verifies the result of Denison and Baum [2]

and previous laboratory measurements which have shown that the response consists generally of a single peak in the range of frequency for which the quasi-steady approximation appears to be accurate. Some negative peaks exist at the other frequencies, implying that the resonance in the gas phase tends to damp the oscillatory motion. At most higher frequencies, the system seems to be unstable.

The real part of the burning rate shows a trend similar to the acoustic admittance at the quasi-steady region, although the magnitude is significantly different. But these trends differ from each other at the higher region. Over $\omega = 100$, the profile tends to have a second mode oscillation as the pressure varies in the flame zone. It must be emphasized that the admittance function alone is not sufficient to describe the stability of the system unless the velocity coupling is concerned.

Temperature distributions are given in Fig. 6. Since the temperature is related to the density, it can be analyzed easily by comparing the results with those in Fig. 5. Accordingly, the distribution of the temperature shows the profile opposed to that of the density. Temperature rises are predominant along the reaction region for $\omega > 1$. Note that no significant temperature changes occur at the interface as imposed by the boundary conditions. Negative amplitude near the flame edge may be caused by the stagnation phenomena in which the density increases. Because of the pressure unity, the imaginary parts of the

temperature are given as reciprocals of those of the density.

The fuel consumption at a given frequency is shown in Fig. 7. Trends for species distributions are opposite to those for temperature distributions in that negative maximum occurs along the reaction region. Note that linearly diminishing the fuel near the flame edge affects the temperature changes slowly (Fig. 6).

The results of the second order perturbation are shown in Figs. 10-28. As previously indicated, each variable of the second order response to acoustic disturbance has two components: one time-dependent component that oscillates at twice the fundamental frequency and one that is time-independent. The latter represents a shift in the mean value, thus causing a shift of the mean burning rate. It should be noted that the variations for the second order system may be characterized by the sum of those two parts, with the factor $e^{i2\omega t}$ ranging between -1 and 1 and zero indicating the case of time-independence.

Figures 10-15 show the distributions in the second order time-independent case. The calculations are also conducted by imposing the pressure of unity at the flame edge as the Dirichlet condition. General trends show that shifts in the mean values are evident.

In Fig. 10, the pressure varies from $\omega = 5$, which is half of that in the first order system. For the higher frequency region, the oscillatory motion is significant,

which may affect the velocity distribution. Figure 11 shows all of the variables for $\omega = 1$, indicating that shifts in mean values may have the affect of damping; however, the opposite may be true for higher frequencies as demonstrated in Figs. 12-15. The trends are roughly similar to those of the first order illustrated in Fig. 4a, but they have different amplitudes because of the nonlinearities in the higher order system. In Figs. 12-15, the distributions of field variables in lower frequencies appear to vary negligibly along the flame thickness, while the opposite is true for $\omega > 1$. In particular, since the pressure varies at $\omega > 5$, the velocity changes significantly at those frequencies, implying that the system is unstable at higher frequencies. As mentioned earlier, a shift of the mean burning rate is the most significant effect in the time-independent system. This result will be discussed in Fig. 28.

The distributions in the second order time-dependent case are shown in Figs. 16-21. Here, the pressure also varies from $\omega = 5$ and oscillates for higher frequencies. Note that the amplitude variations are smaller than those in the time-independent case. The trends seem to reduce the total effect of the second order system.

Figures 22-27 show the variations of field variables in the second order system by summing time-independent and time-dependent effects, with the factor $e^{i2\omega t} = 1$ for time-independent effects. Note that amplitude variations of each

variable in the flame zone are significant at $\omega = 1$, which is the same result obtained for the case of the first order system. Finally, Fig. 28 reveals that the shift in burning rate versus frequency is consistently negative, as asserted by other investigators. The offset is relatively small in the quasi-steady region, but increases with oscillatory motion along the frequencies. The variations of the acoustic admittance are similar to those of the first order system, except that oscillatory deviations are more predominant at higher frequencies in both time-independent and time-dependent cases.

Parameter studies are conducted for the first order and are summarized as follows. Decreasing the density ratio β affects the variables shifted slightly to the negative direction, keeping the distribution profiles constant. Changing the latent heat of solid L exerts a negligible effect on the variables, but a very small value of L shifts the system toward instability. Increasing the surface activation energy E_s or the gas phase activation energy E_g reduces the magnitude of the variables, keeping the same profiles. Changes of the rate constant and viscosity effect coefficient strongly affect the system, such that every aspect discussed herein will change.

The present study could be extended to the multi-dimensional case by introducing the appropriate axial mean flow field. It is well recognized that fluctuation of the gas velocity parallel to the propellant surface affects the

burning rate in terms of velocity coupling; therefore, this quantity must be considered together with pressure coupling for a satisfactory measure of stability.

A simple calculation has been accomplished using the artificial axial flow velocity. However, difficulties of the boundary conditions could not be eliminated. A test run shows that the existence of a small amount of the axial flow reduces differences between the variable amplitudes in each frequency considered, thus leading the system toward stability.

CHAPTER III

CONCLUSION

A multi-dimensional numerical model for the premixed flame acoustic instability is proposed and solved using the finite element method. The governing equations are perturbed to the second order and formulated with the Galerkin finite elements. The results have direct bearing on the validity of published theories of solid propellant combustion instability at the lower frequency region where the uniform pressure assumption is valid. Extended studies are made on the higher frequency region and some new results are obtained.

Under the restricted boundary conditions, the following conclusions, based on numerical computations, are reached:

- (1) The stability characteristics for the low frequency range in the first order system have been verified to be the same as those reported in the literature. For example, the acoustic admittance is controlled by the burning rate, negative for low frequencies, whereas the opposite is true for high frequencies.
- (2) Instabilities are likely to occur at high frequencies ($\omega > 10$).
- (3) For the second order system, the trend is similar to the first order system for low frequencies, but

instabilities may appear at frequencies lower than those of the first order system.

- (4) The most significant effect of the second order system is that the admittance is extremely oscillatory between $\omega = 1$ and $\omega = 10$.

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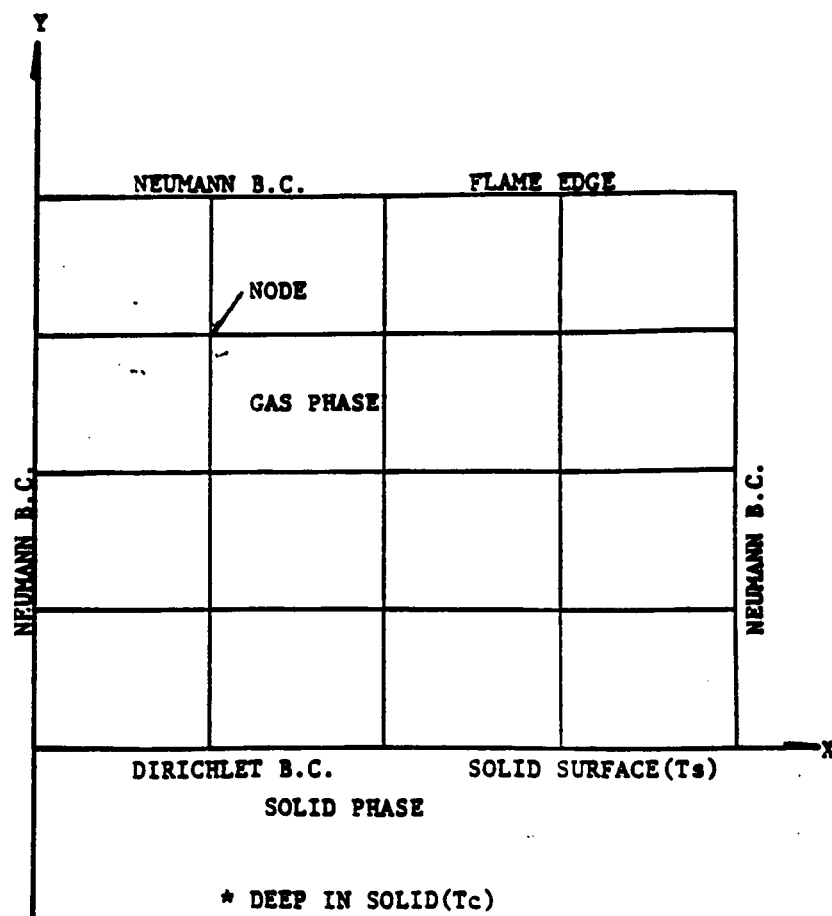


Fig. 1 Finite element geometry.

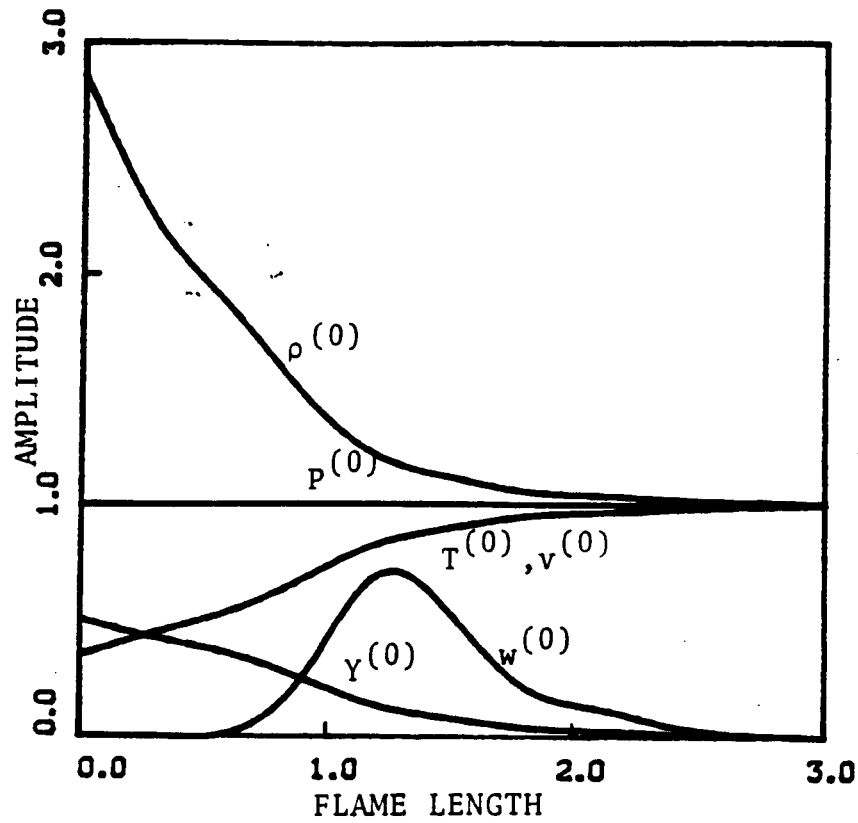


Fig. 2 Steady-state distributions of field variables in flame zone. Parameters used in the calculations are given in Table 1, including $Pr=1$ and $Mb=0.003$. Reference values for non-dimensionalization are chosen from the flame edge. (ρ :density, T :temperature, Y :species of fuel, v :velocity, P :pressure, and w :reaction rate).

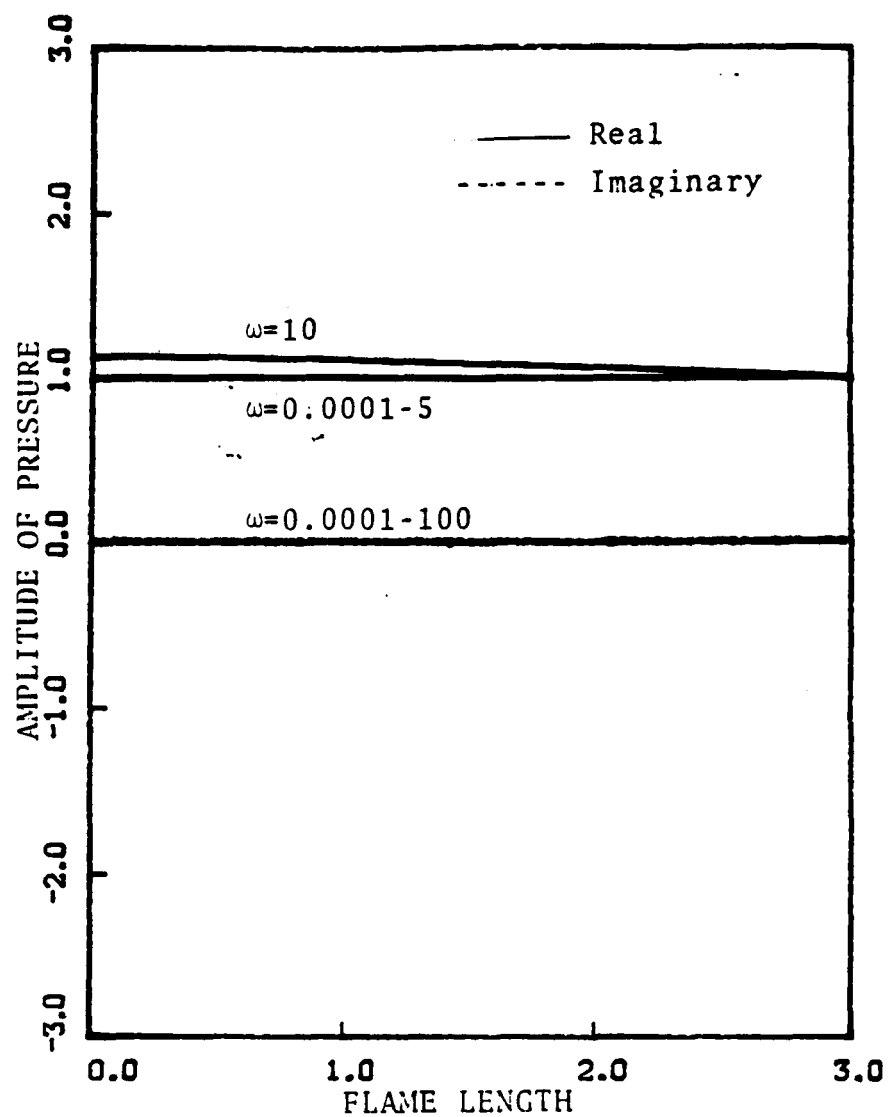


Fig. 3 First order pressure distributions vs. frequency(Dirichlet condition is imposed at the flame edge).

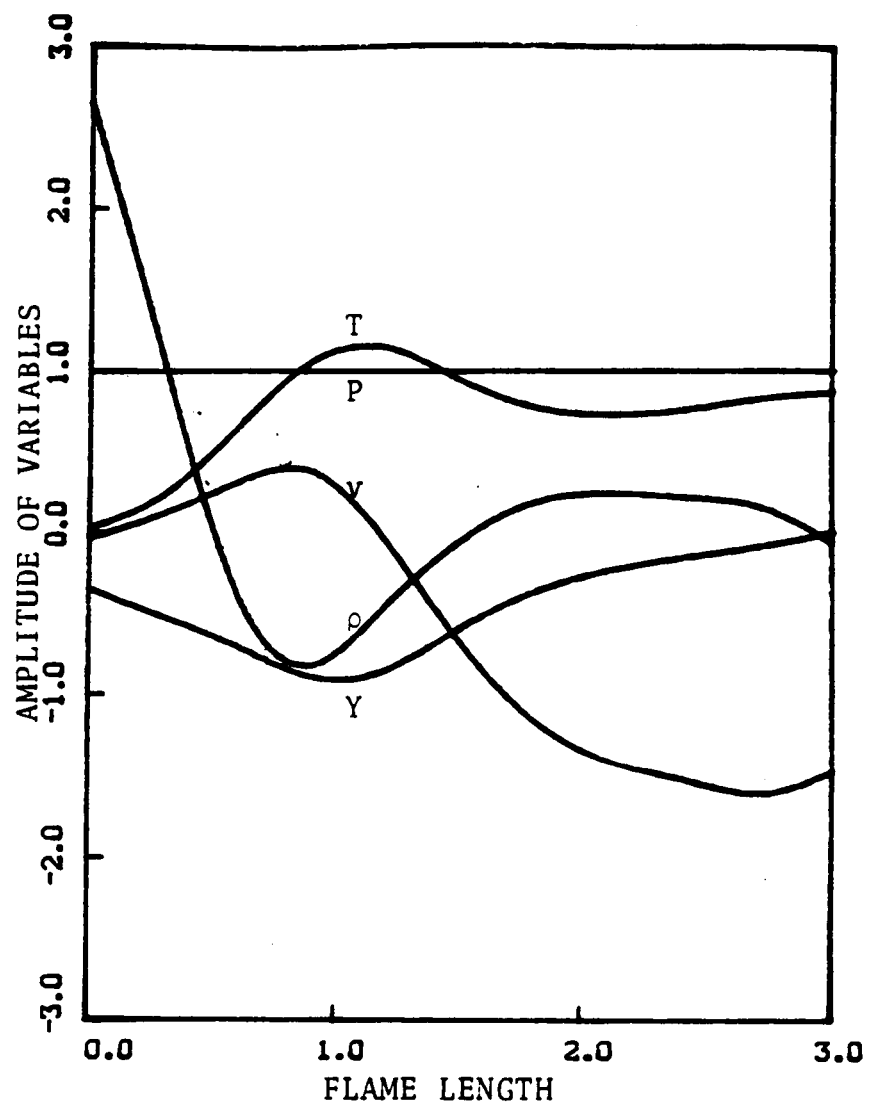


Fig. 4a First order distributions of field variables at $\omega=1$ (real parts).

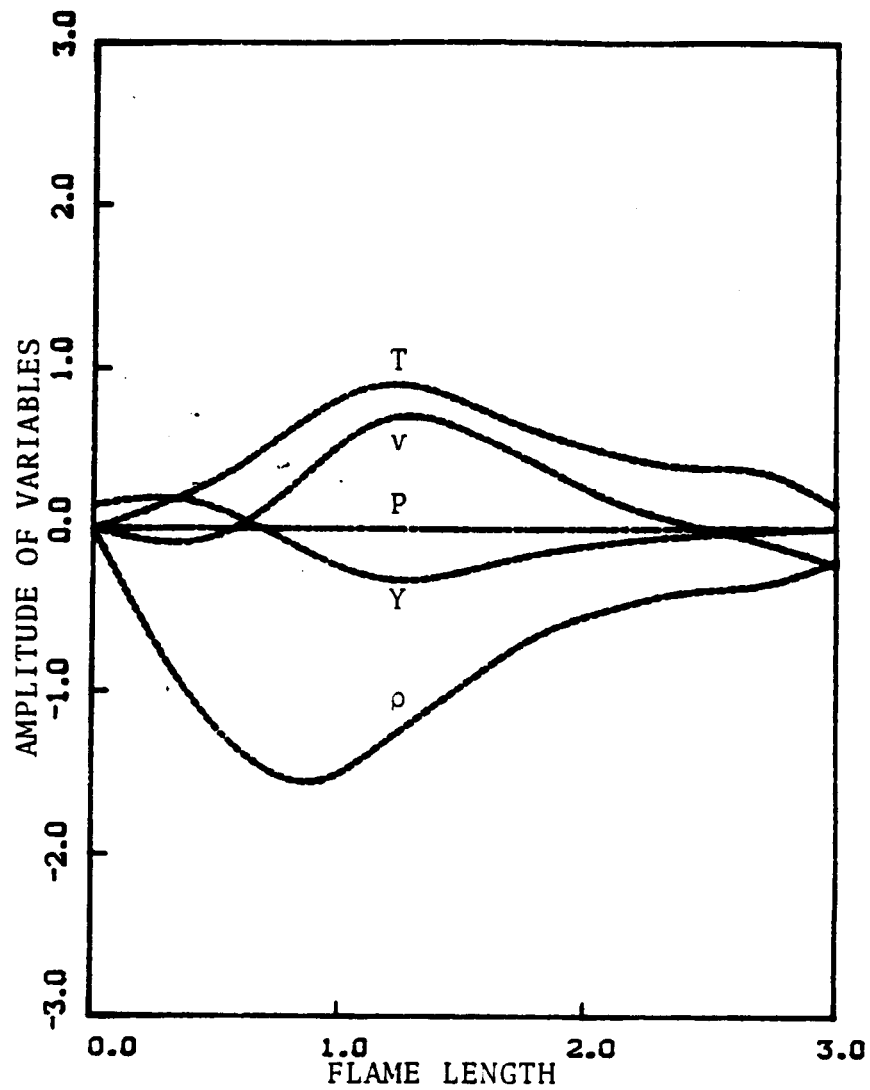


Fig. 4b First order distributions of field variables at $\omega = 1$ (imaginary parts).

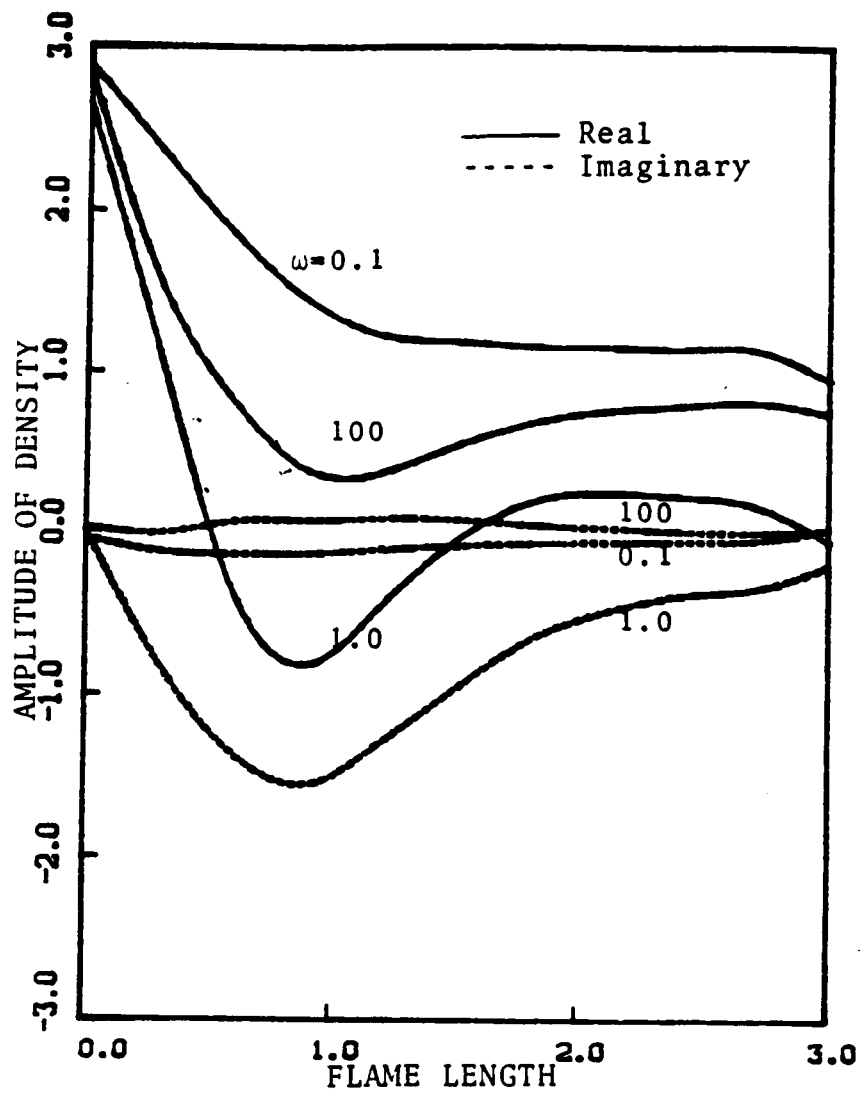


Fig. 5 First order density distributions vs. frequency.

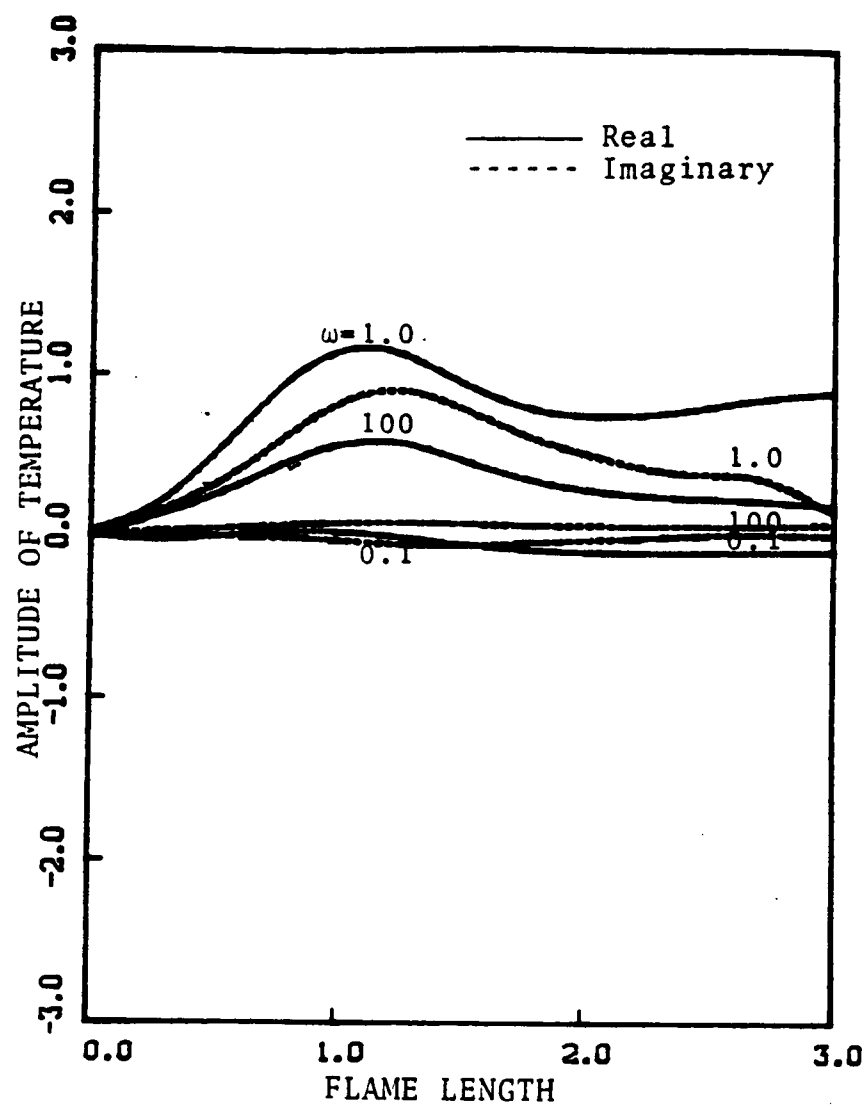


Fig. 6 First order temperature distributions vs. frequency.

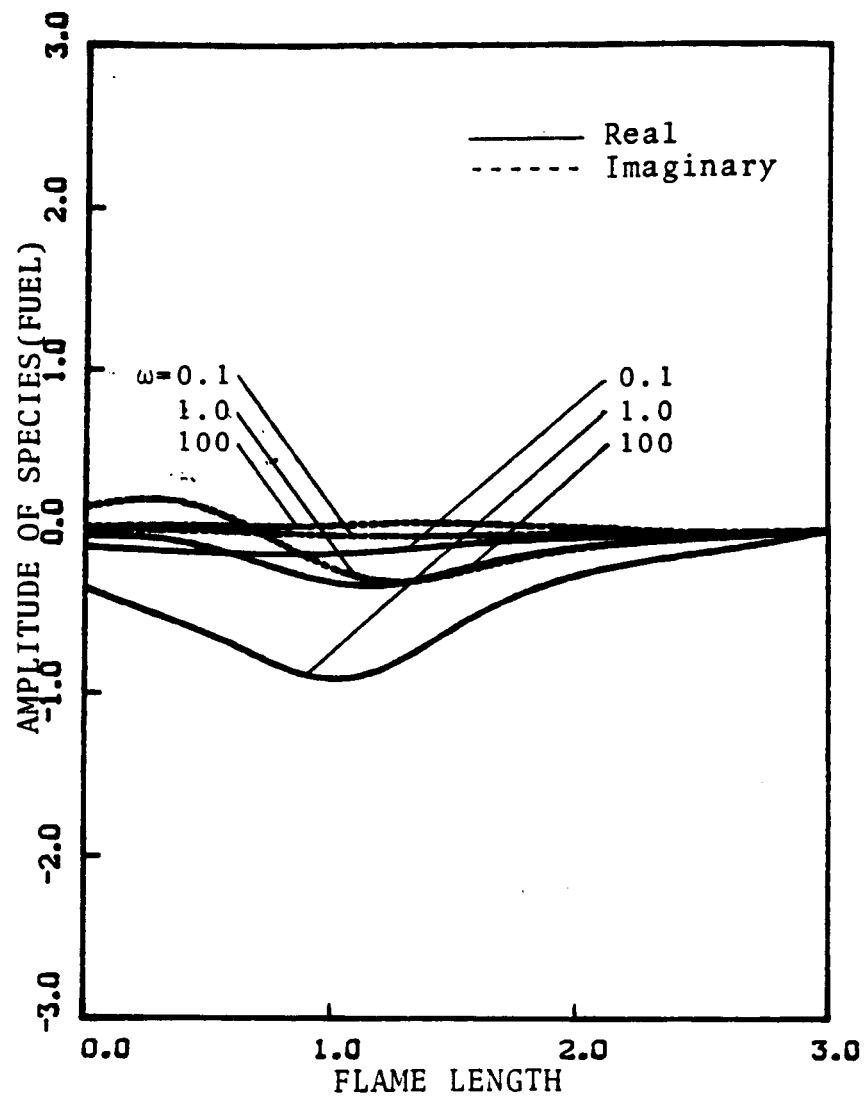


Fig. 7 First order species(fuel) distributions vs. frequency.

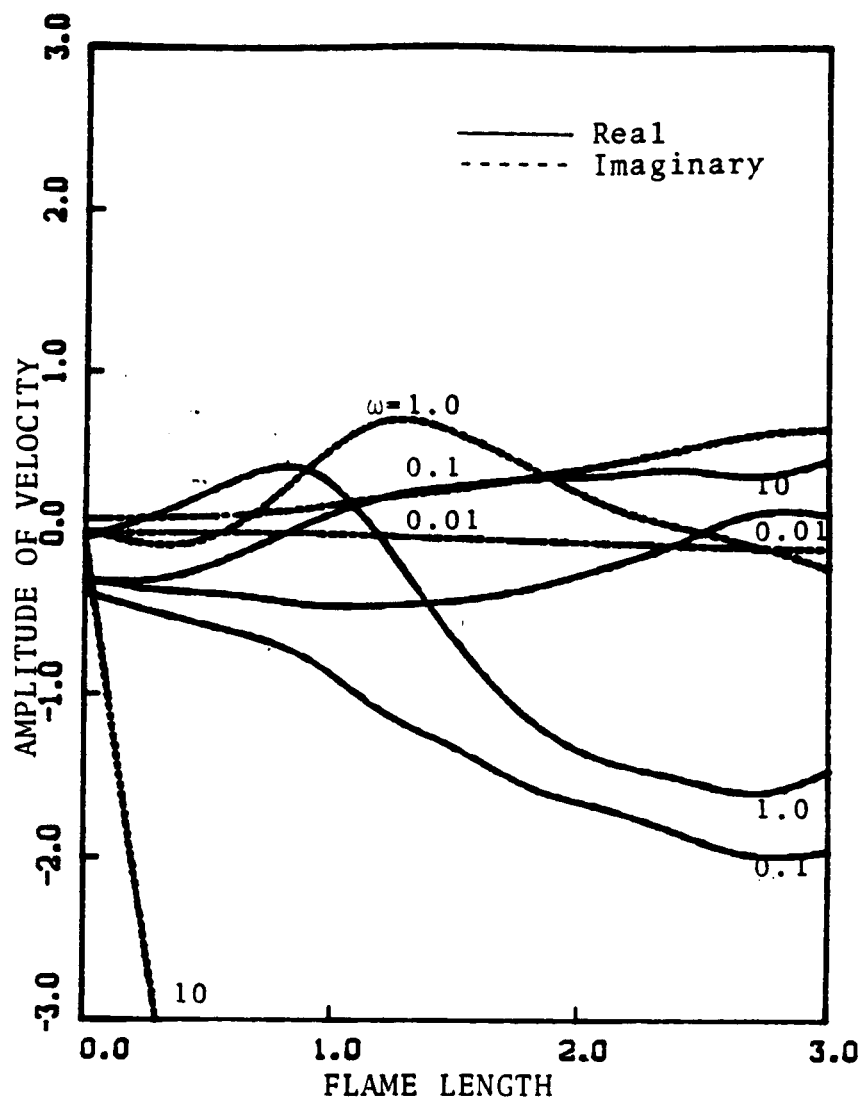


Fig. 8 First order velocity distributions vs. frequency.

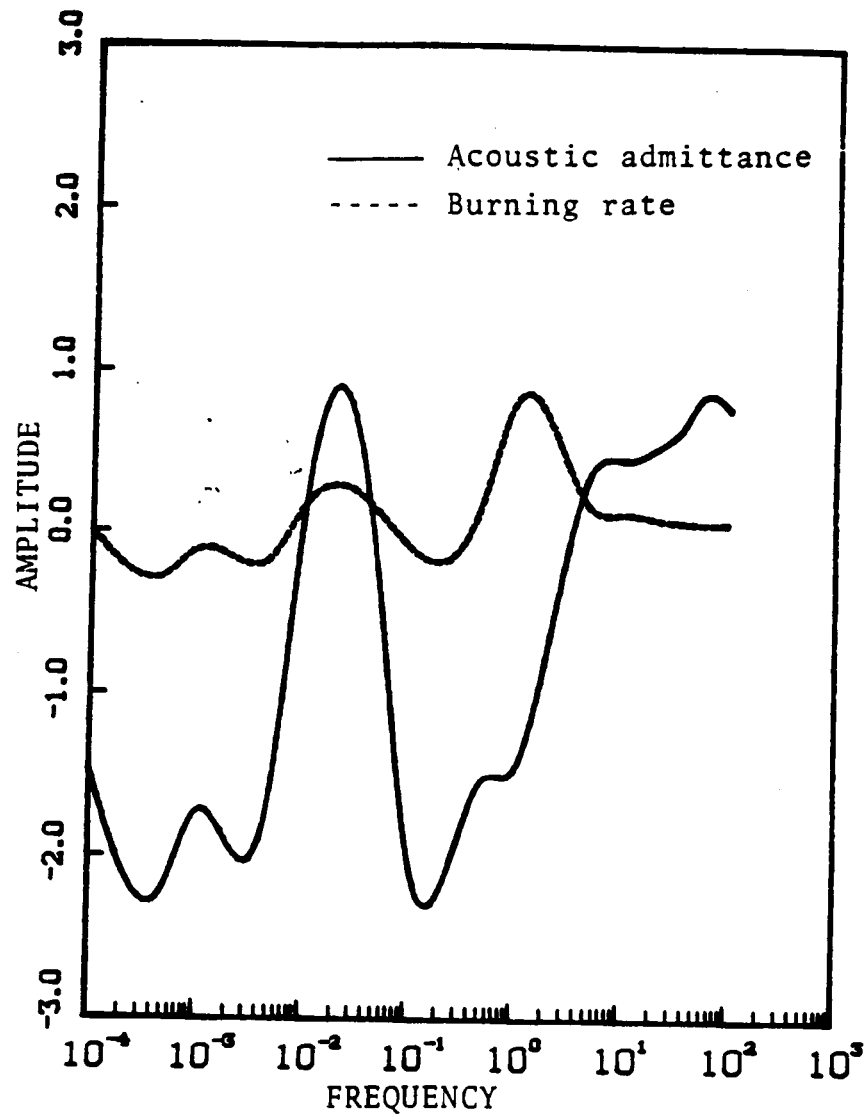


Fig. 9a First order acoustic admittance and burning rate vs. frequency. Positive peak near $\omega = 0.01$ resembles the results of Denison and Baum(2) and T'ien(4).

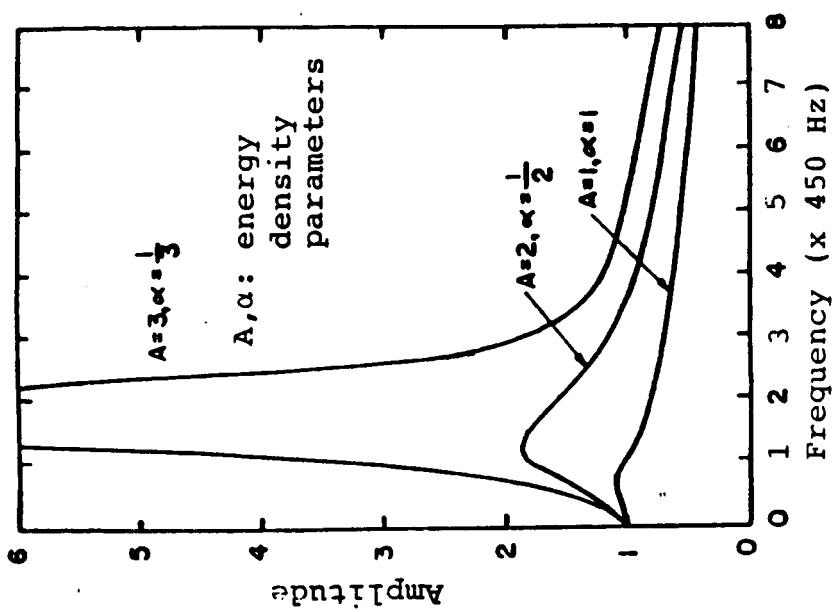


Fig. 9b Steady oscillatory mode: typical dependence of amplitude on frequency of pressure oscillation [Denison and Baum, 1961].

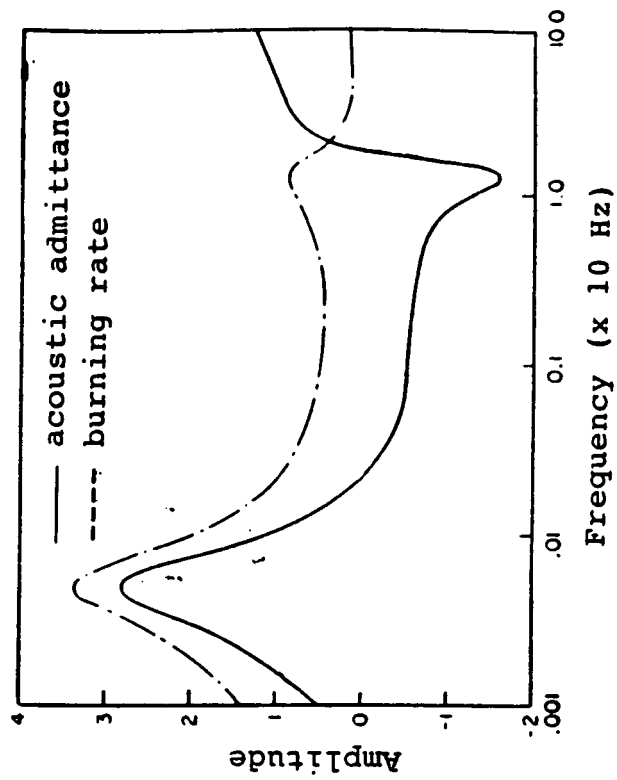


Fig. 9c Real parts of acoustic admittance and burning rate vs. frequency [T'ien, 1972].

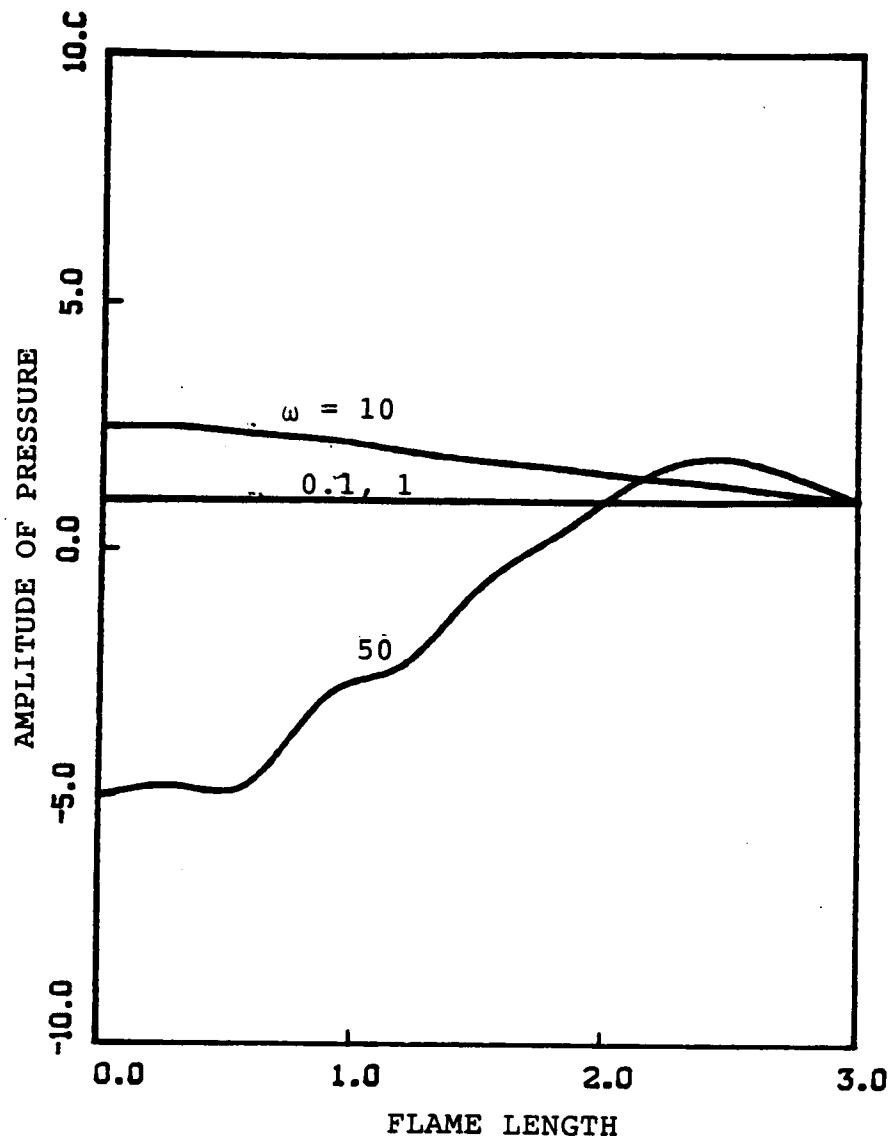


Fig. 10 Second order time-independent pressure distributions vs. frequency.

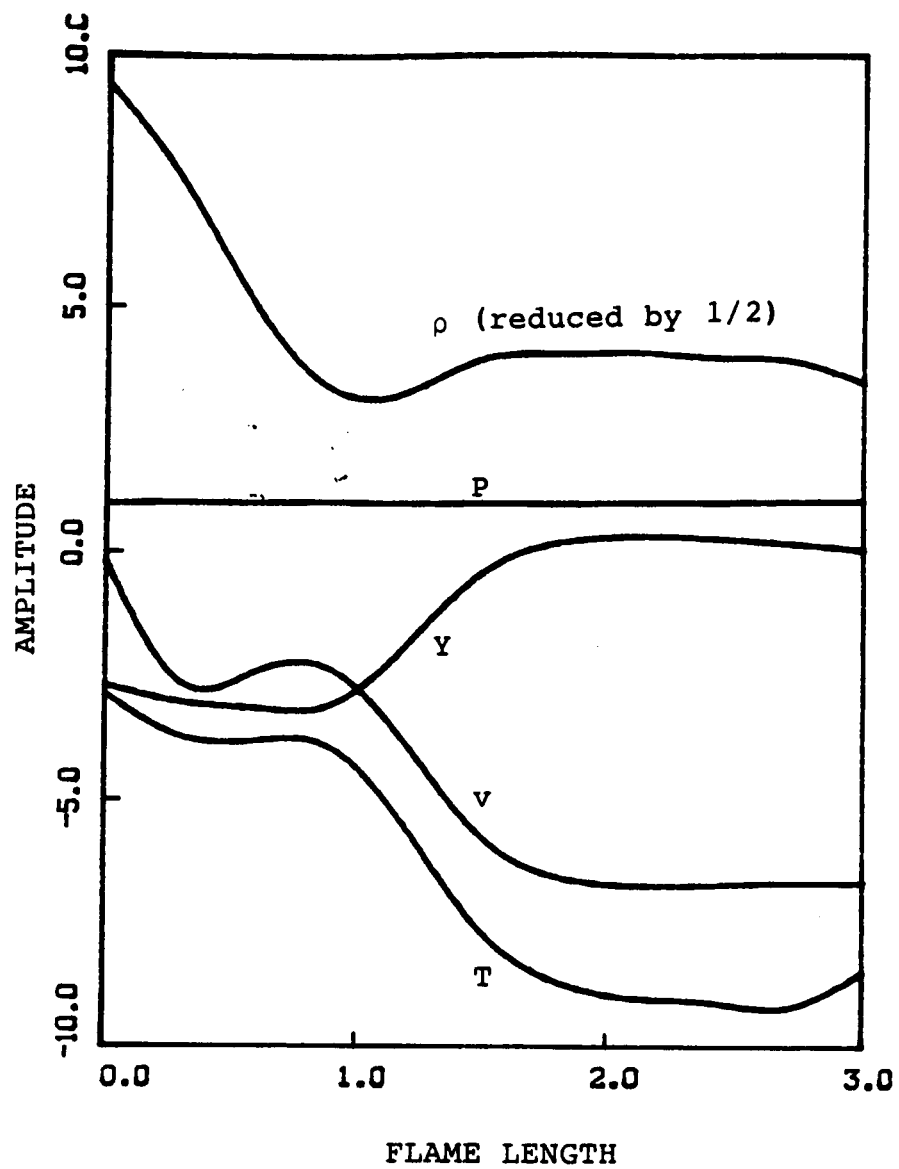


Fig. 11 Second order time-independent distributions of field variables at $\omega = 1$.

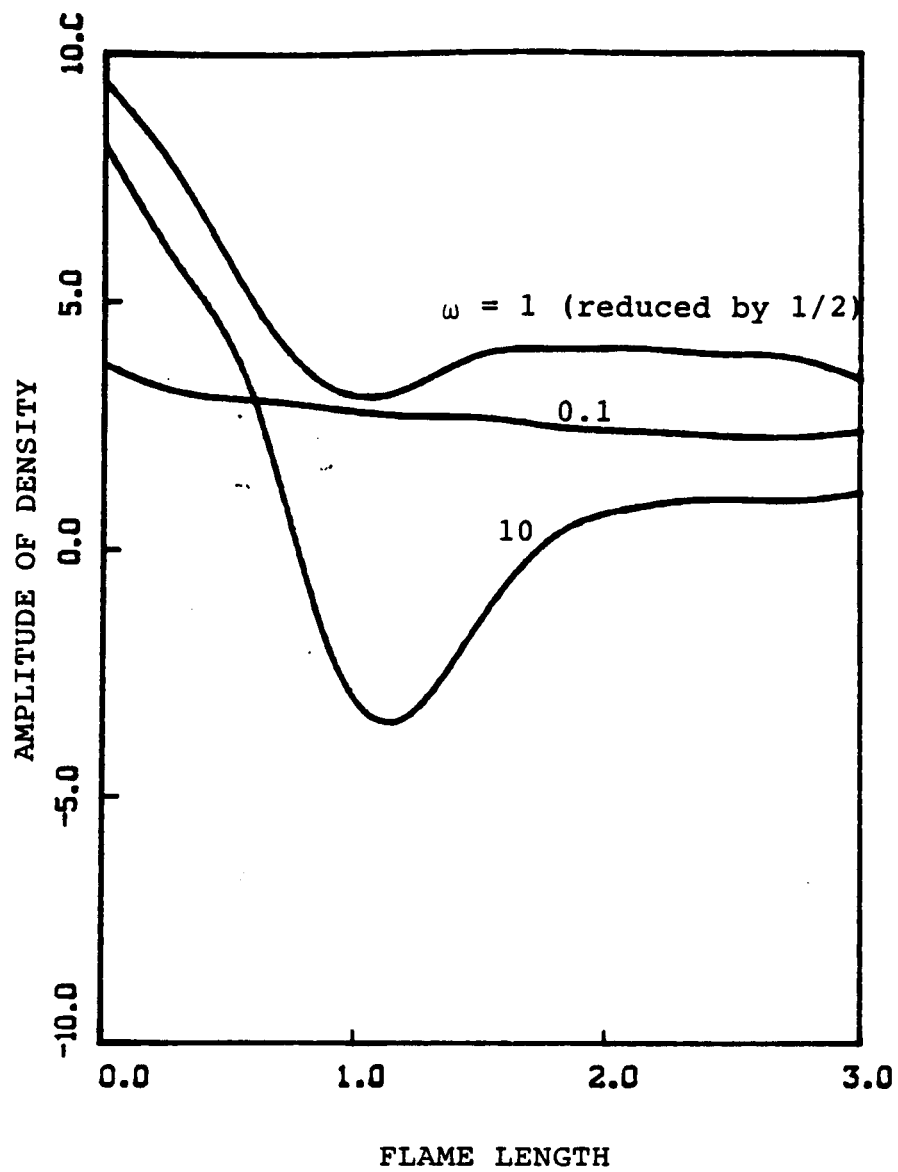


Fig. 12 Second order time-independent density distributions vs. frequency.

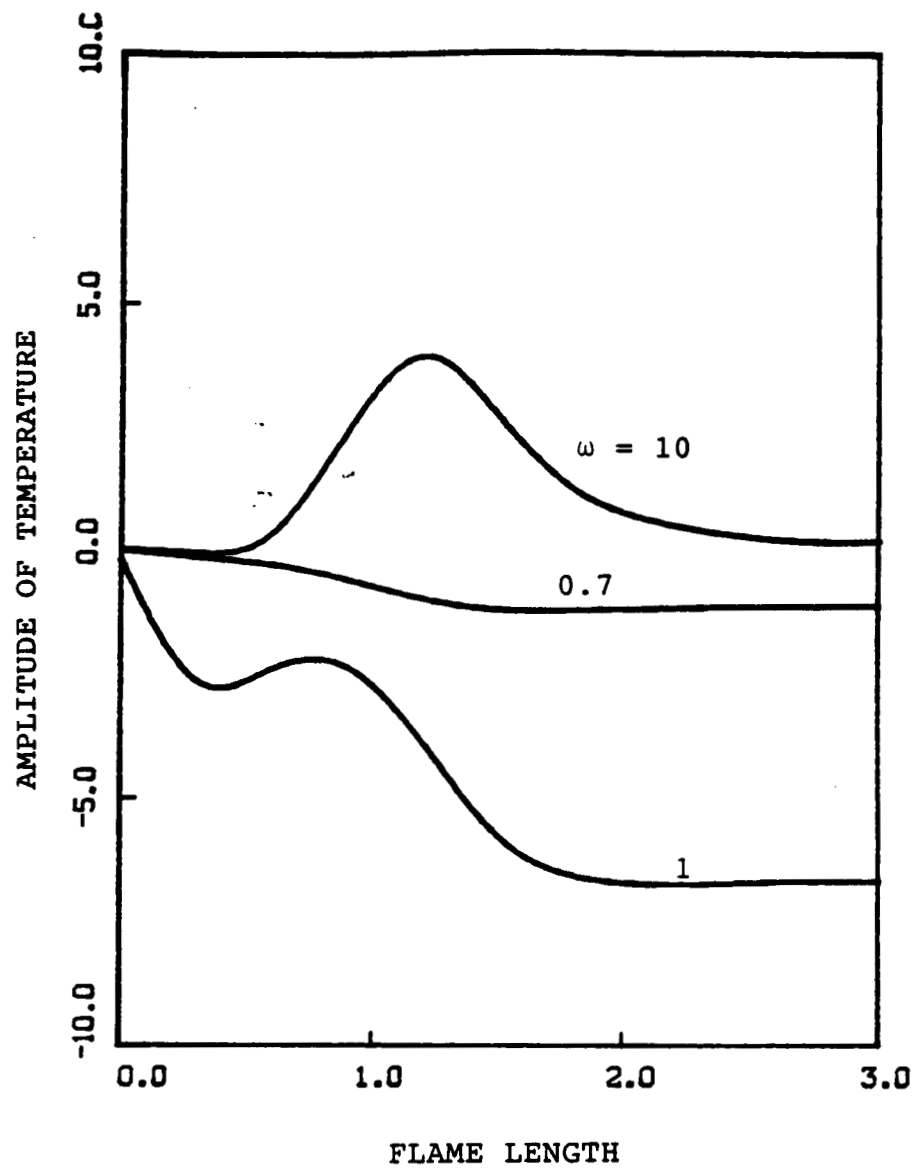


Fig. 13 Second order time-independent temperature distributions vs. frequency.

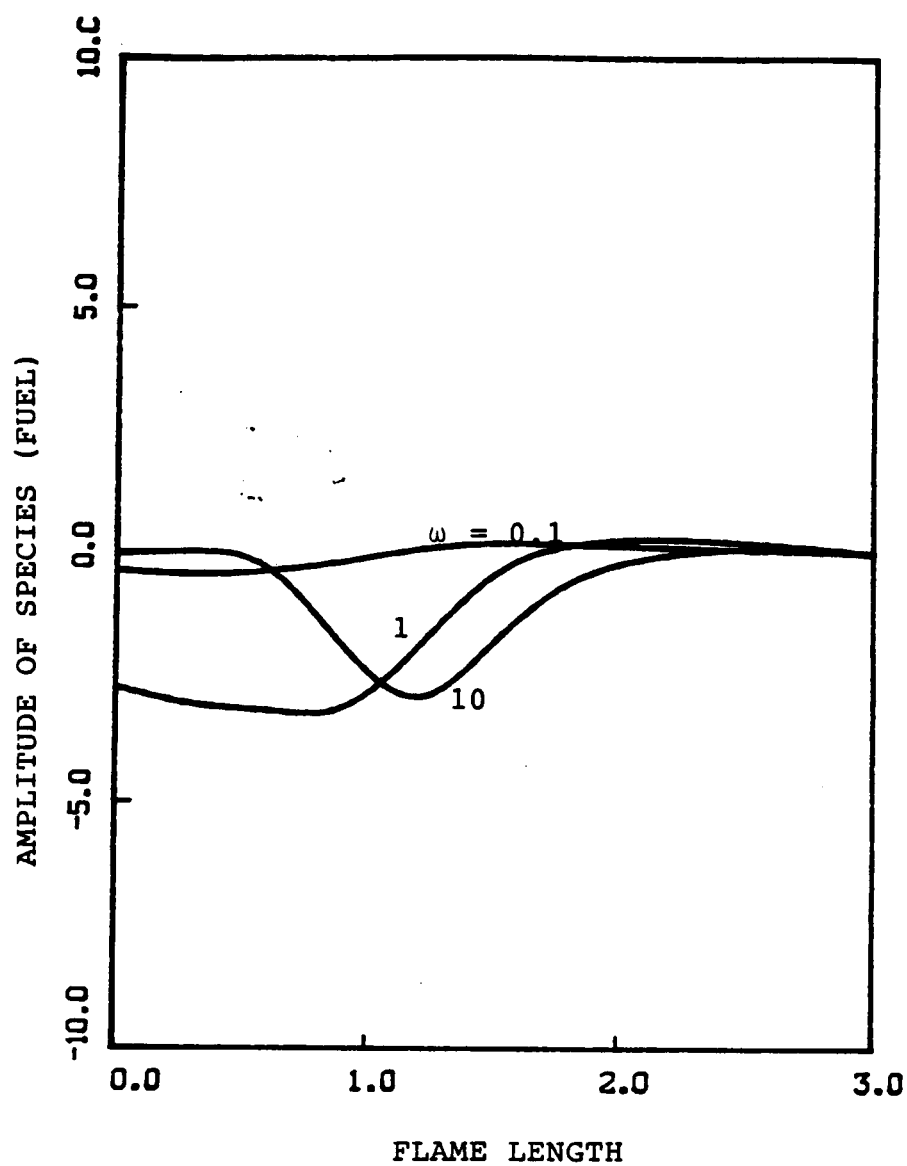


Fig. 14 Second order time-independent species (fuel) distributions vs. frequency.

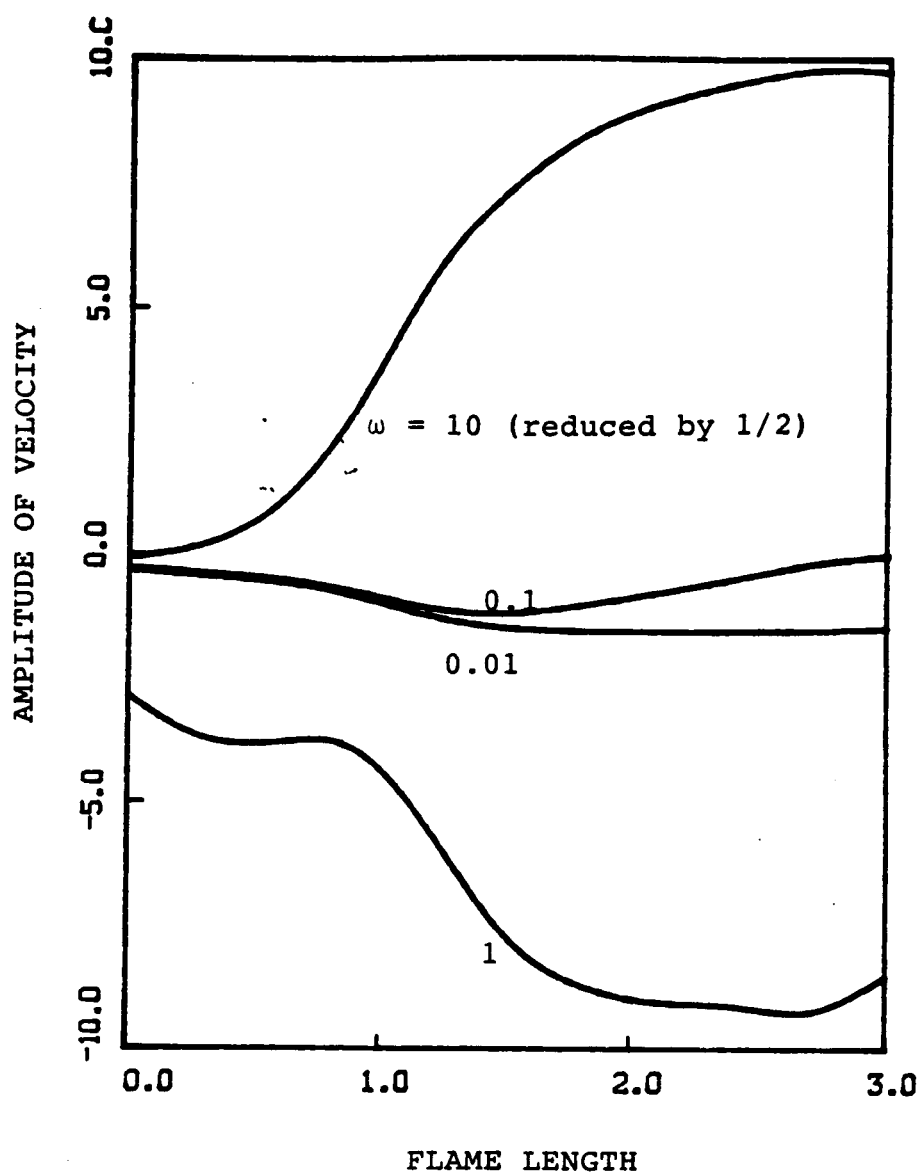


Fig. 15 Second order time-independent velocity distributions vs. frequency.

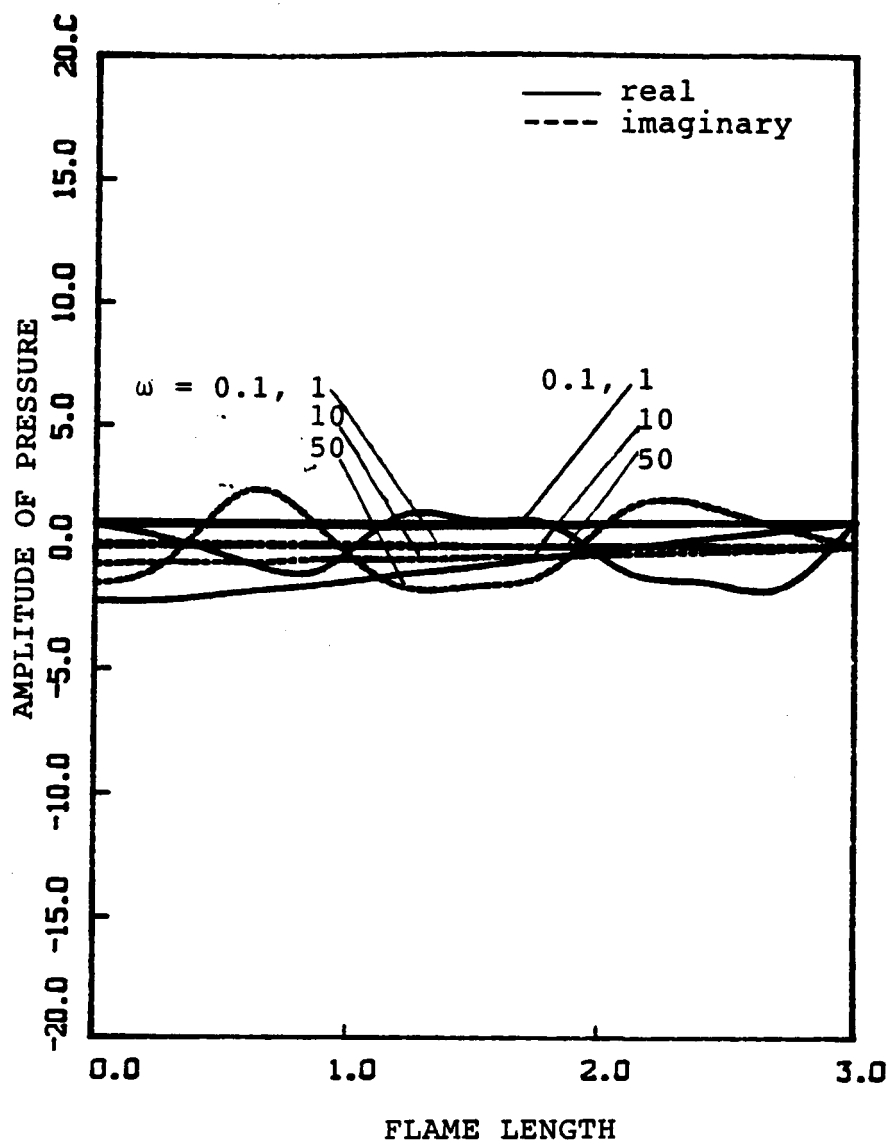


Fig. 16 Second order time-dependent pressure distributions vs. frequency.

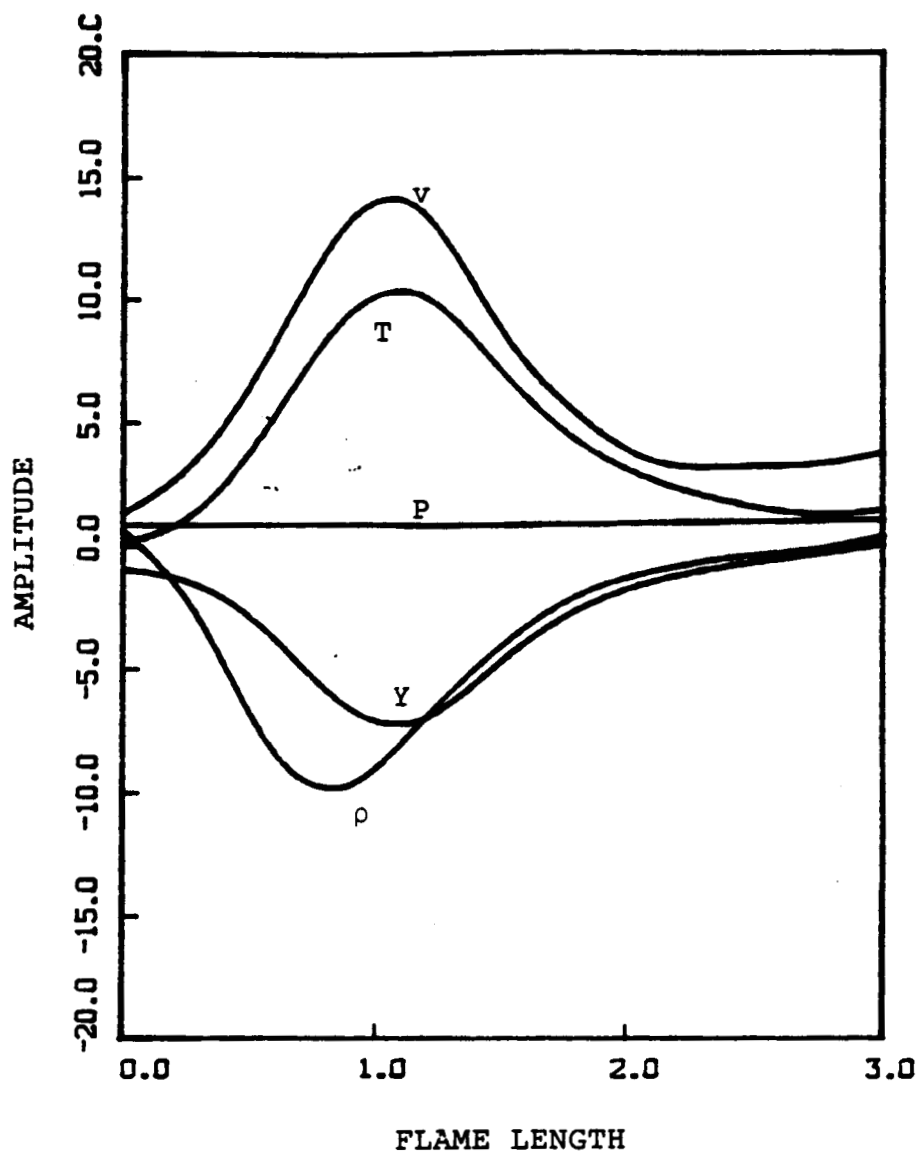


Fig. 17a Second order time-dependent distributions of field variables at $\omega = 1$ (real parts).

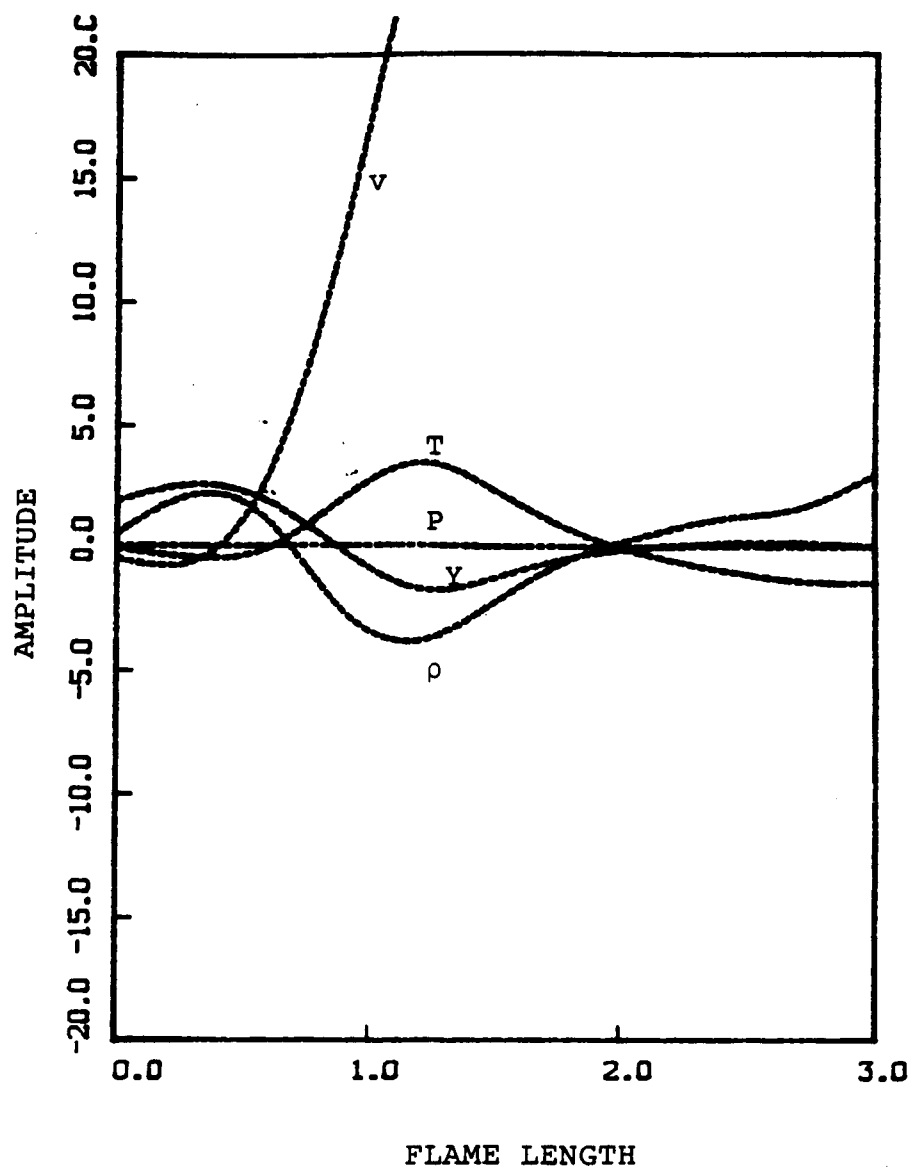


Fig. 17b Second order time-dependent distributions of field variables at $\omega = 1$ (imaginary parts).

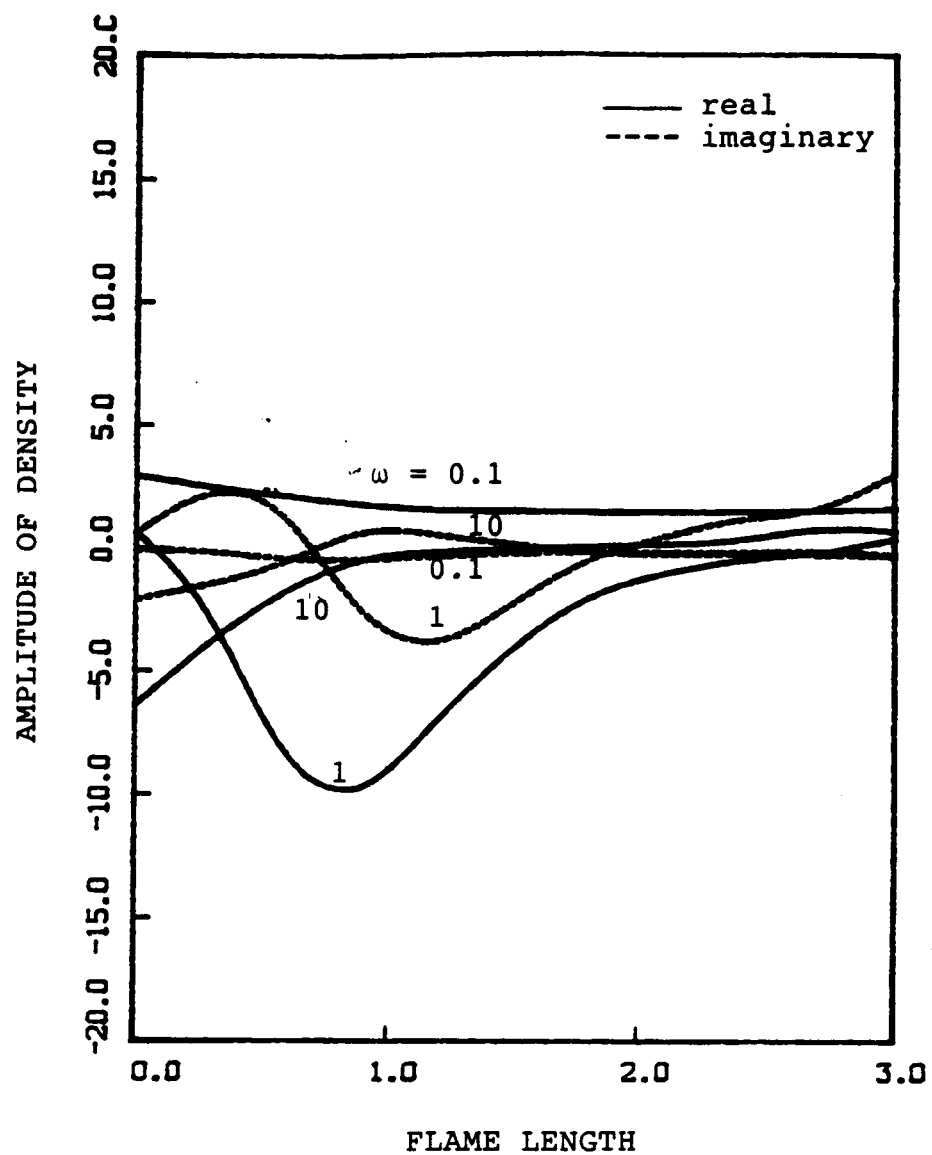


Fig. 18 Second order time-dependent density distributions vs. frequency.

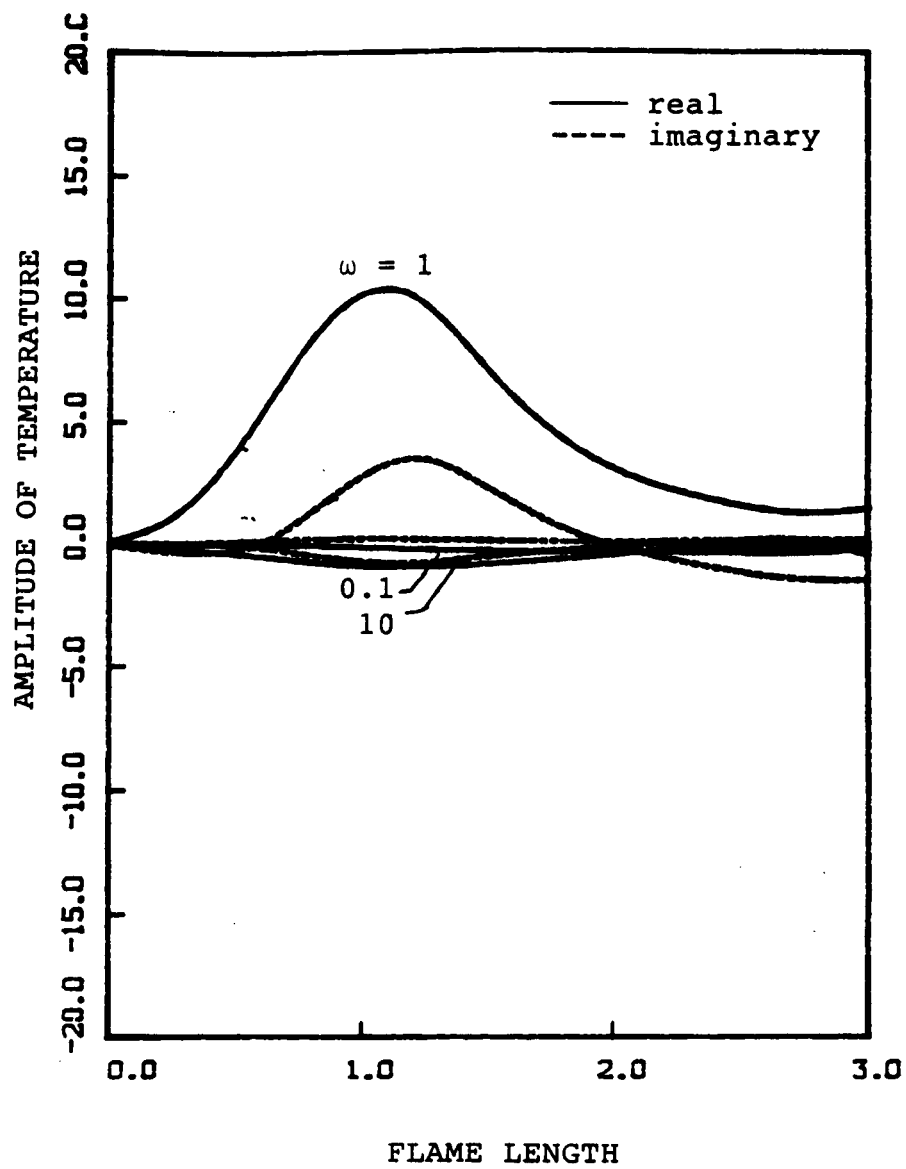


Fig. 19 Second order time-dependent temperature distributions vs. frequency.

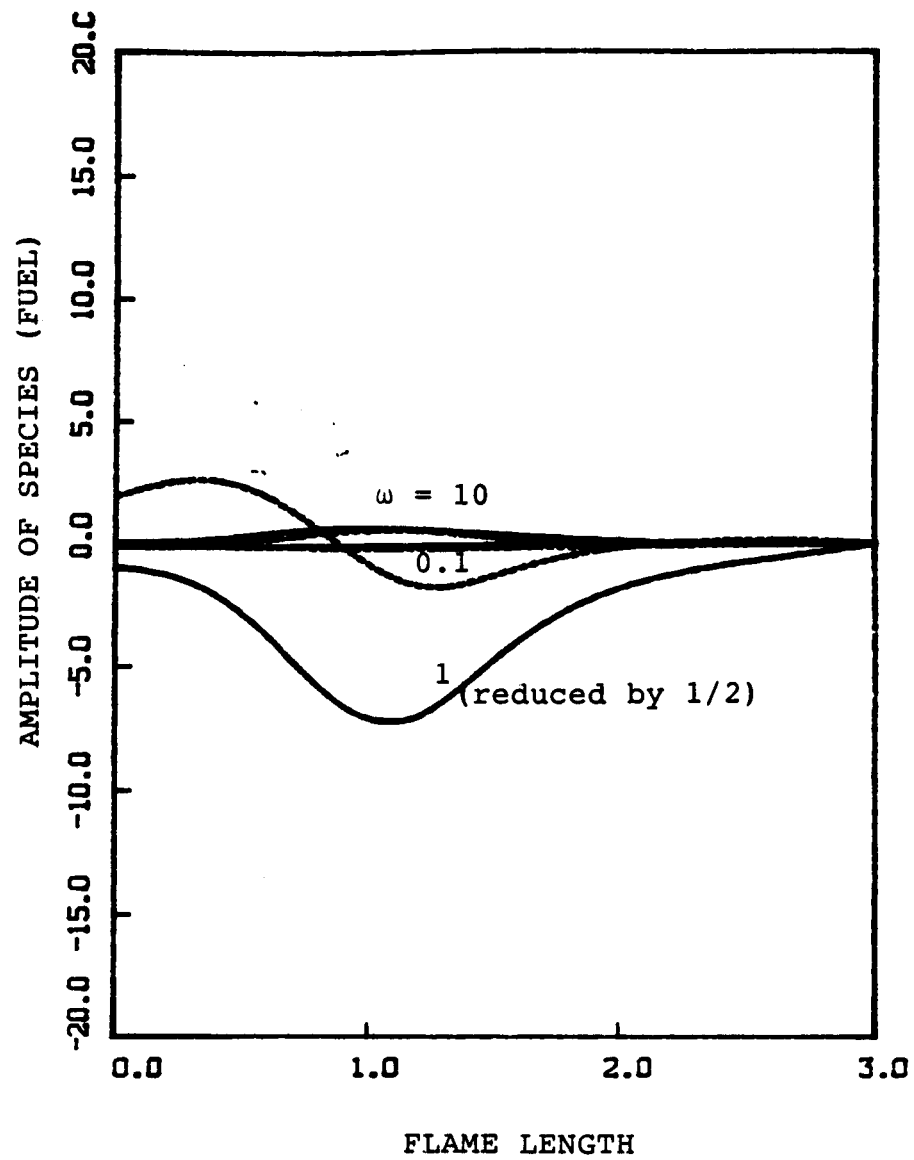


Fig. 20 Second order time-dependent species (fuel) distributions vs. frequency.

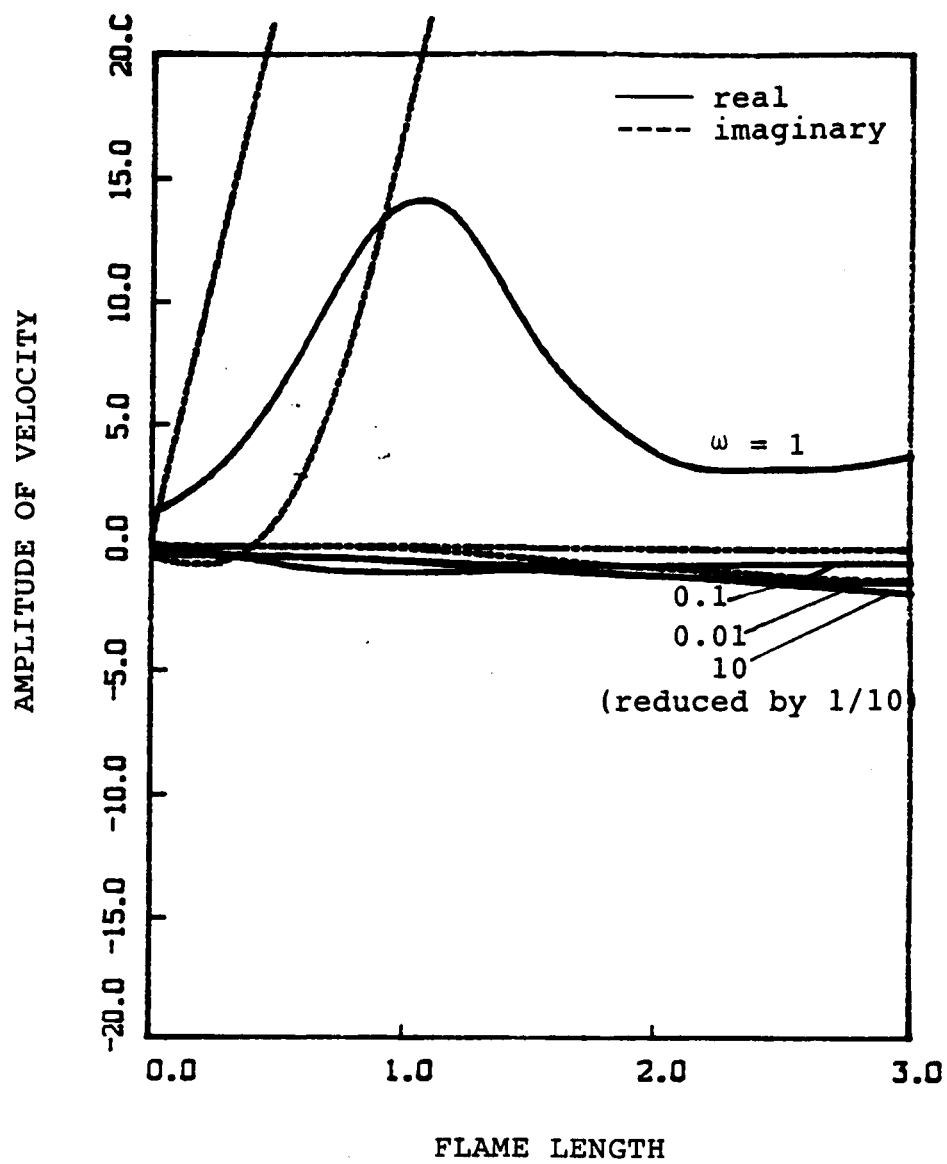


Fig. 21 Second order time-dependent velocity distributions vs. frequency.

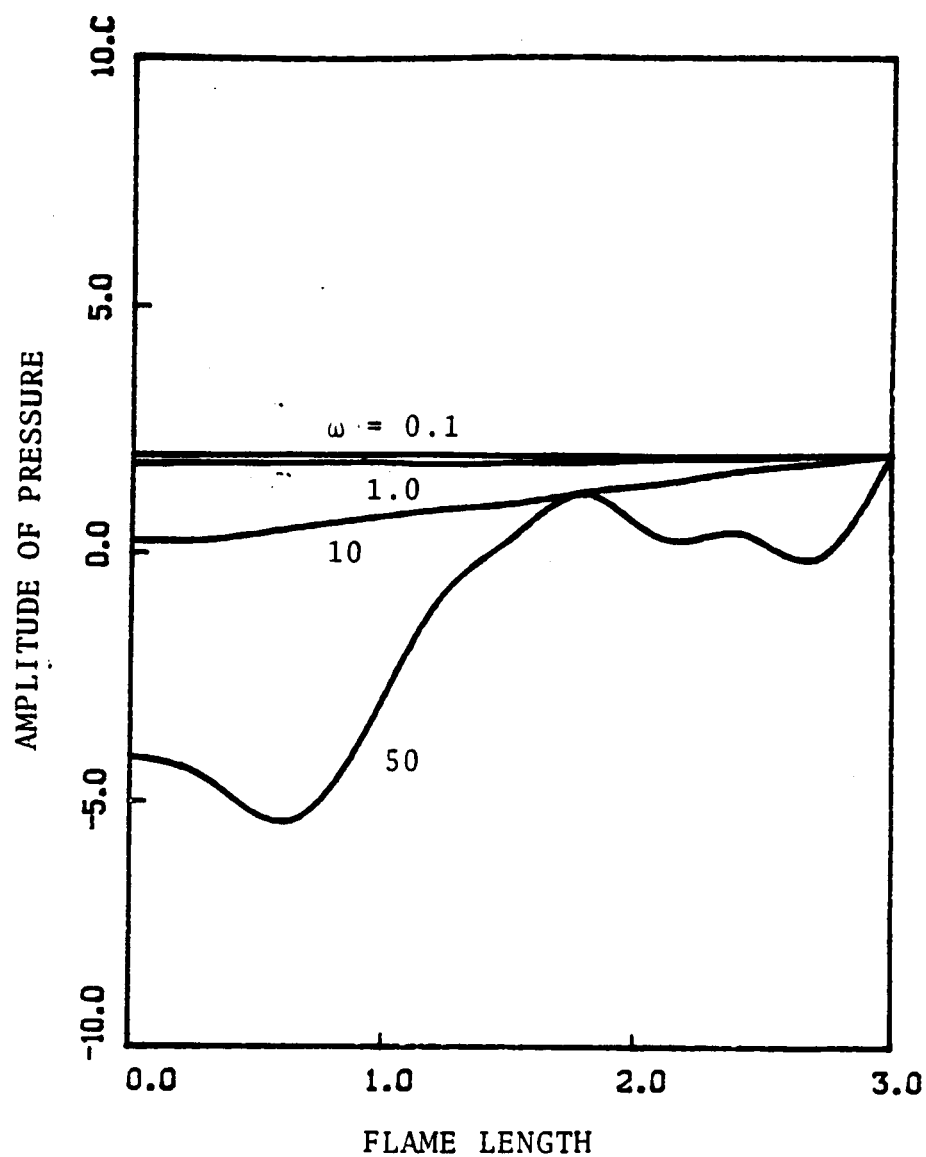


Fig. 22 Second order pressure distributions vs. frequency. Both time-dependent and time-independent coefficients are added for denoting the maximum deviation from mean value.

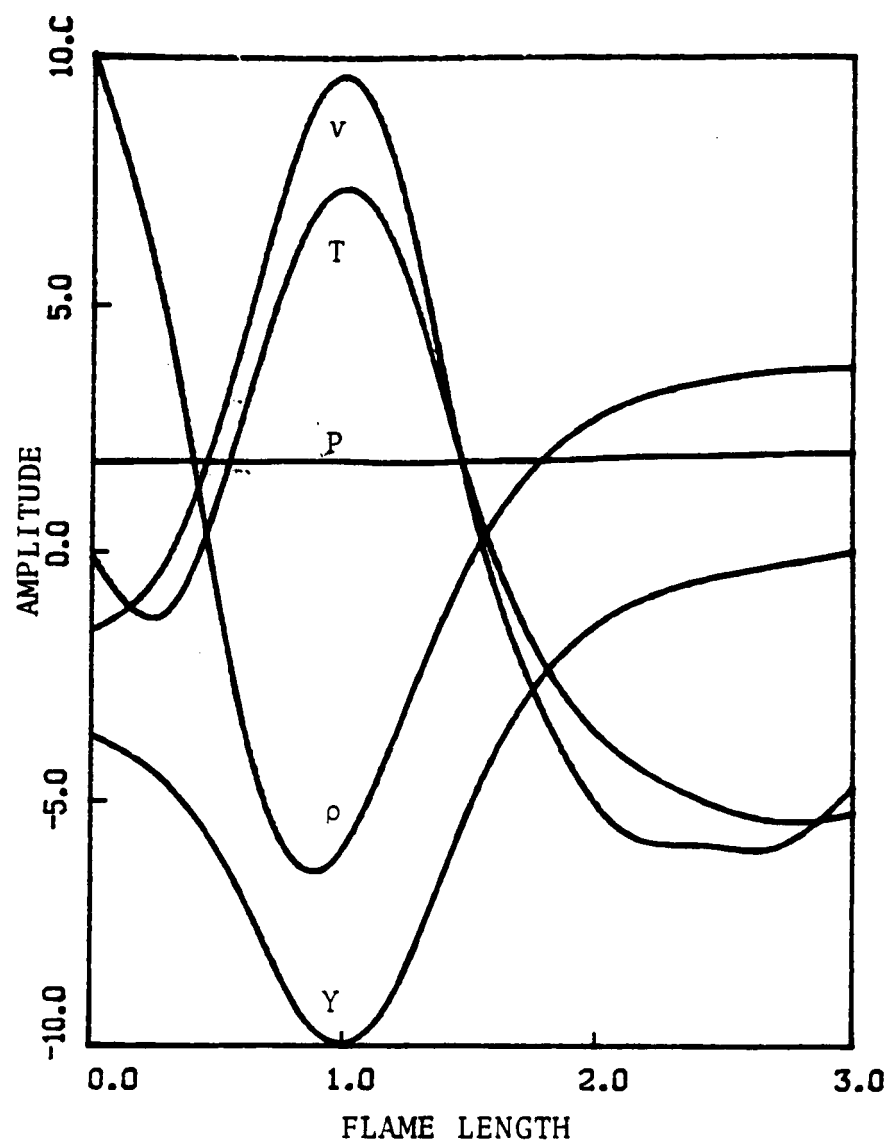


Fig. 23 Second order distributions of field variables at $\omega = 1$.

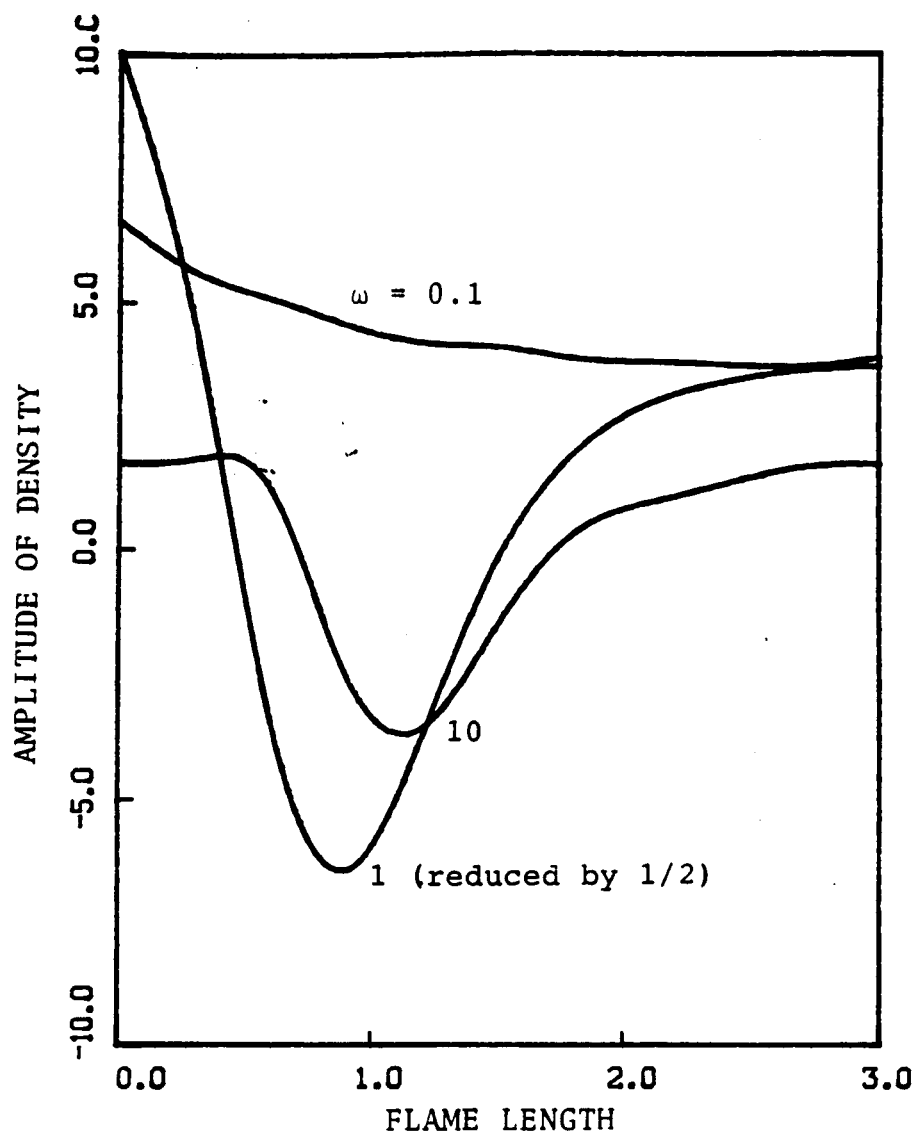


Fig. 24 Second order density distributions vs. frequency.

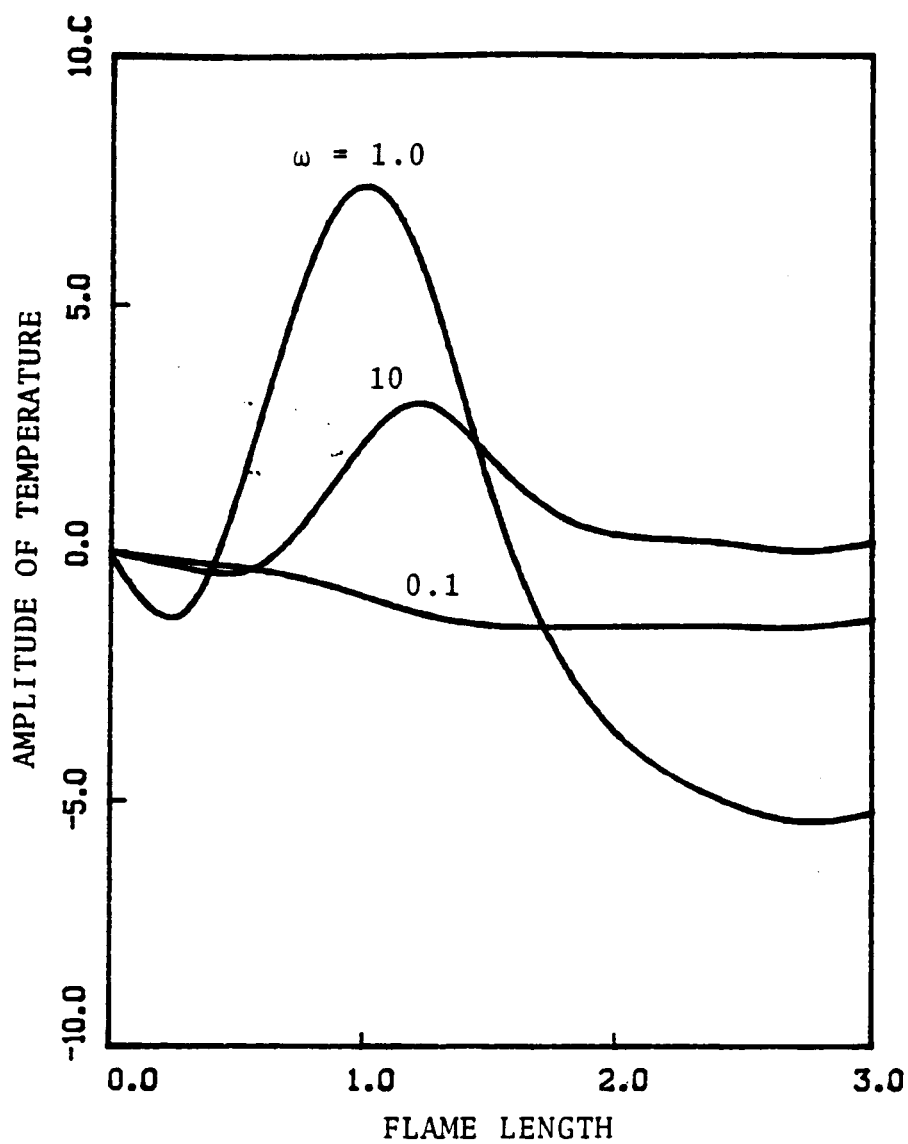


Fig. 25 Second order temperature distributions vs. frequency.

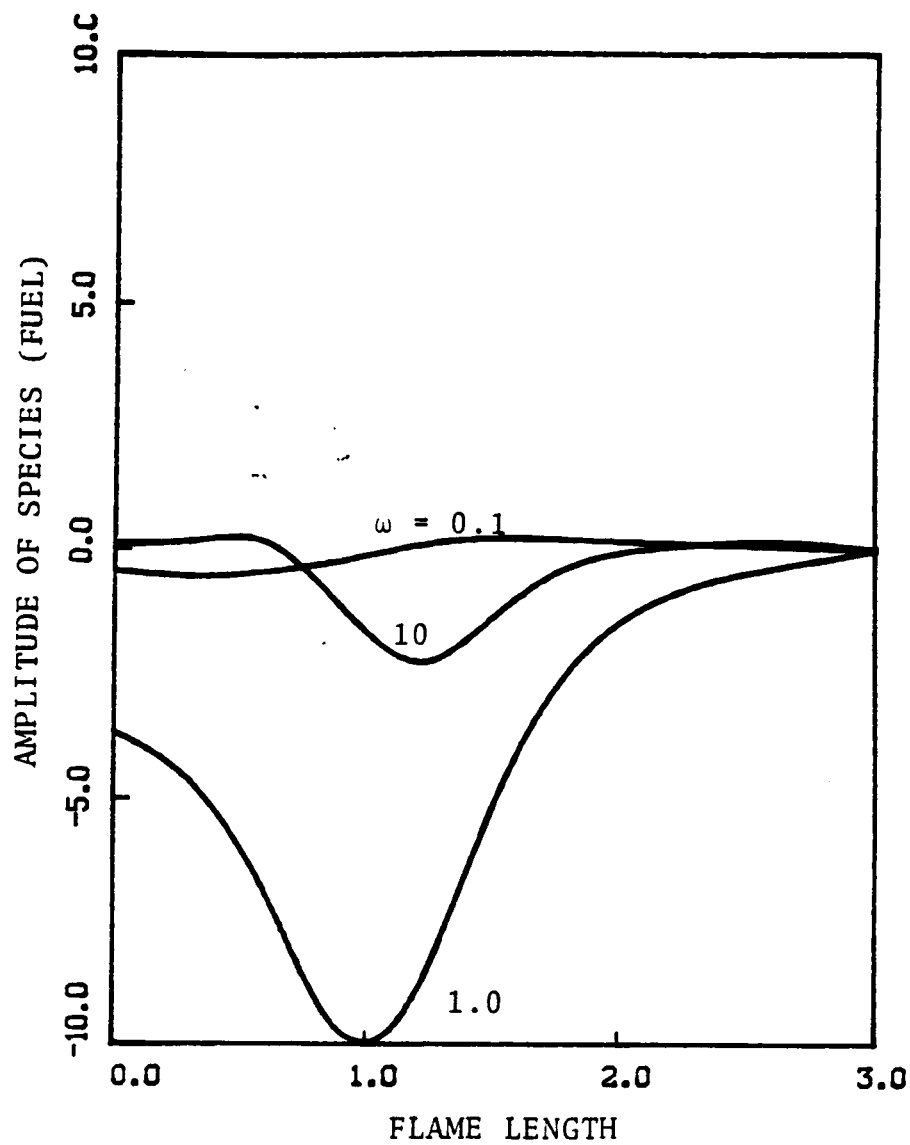


Fig. 26 Second order species (fuel) distributions vs. frequency.

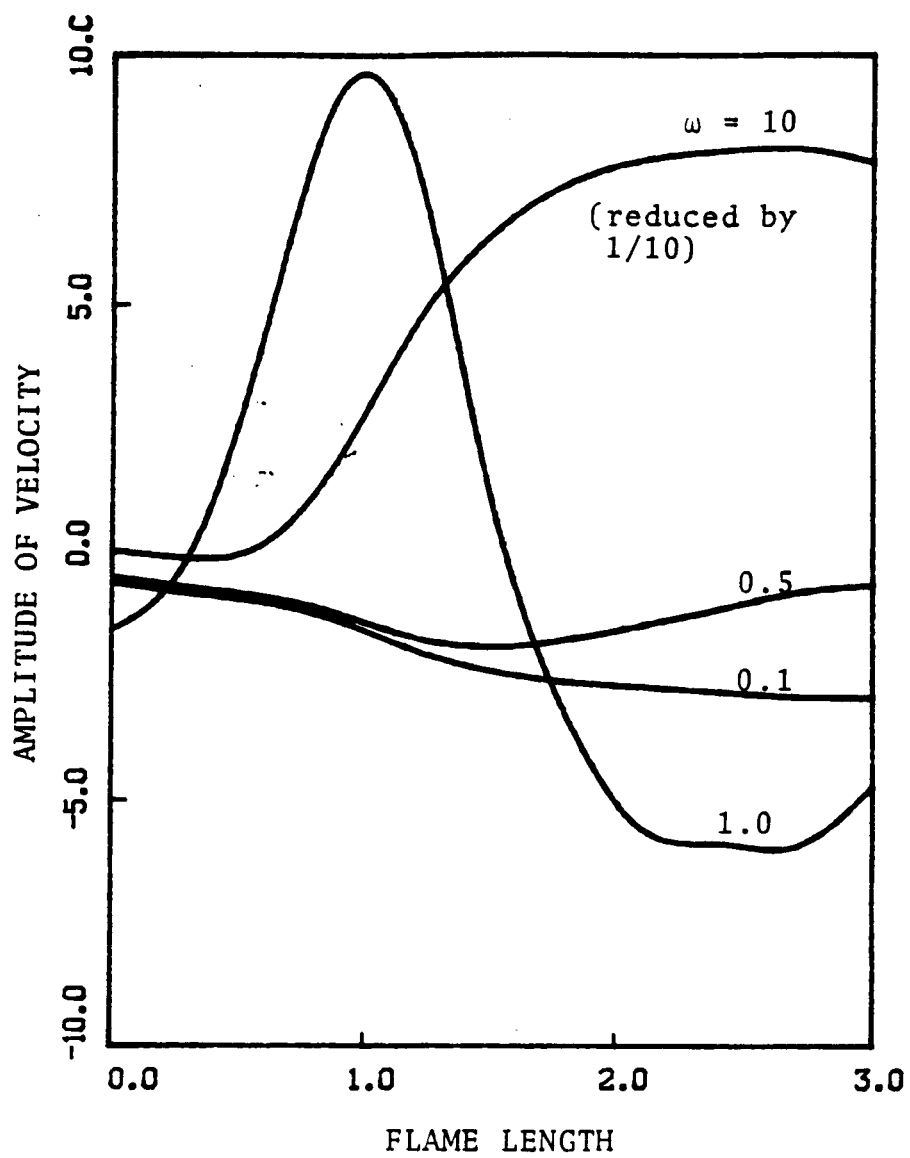


Fig. 27 Second order velocity distributions vs. frequency.

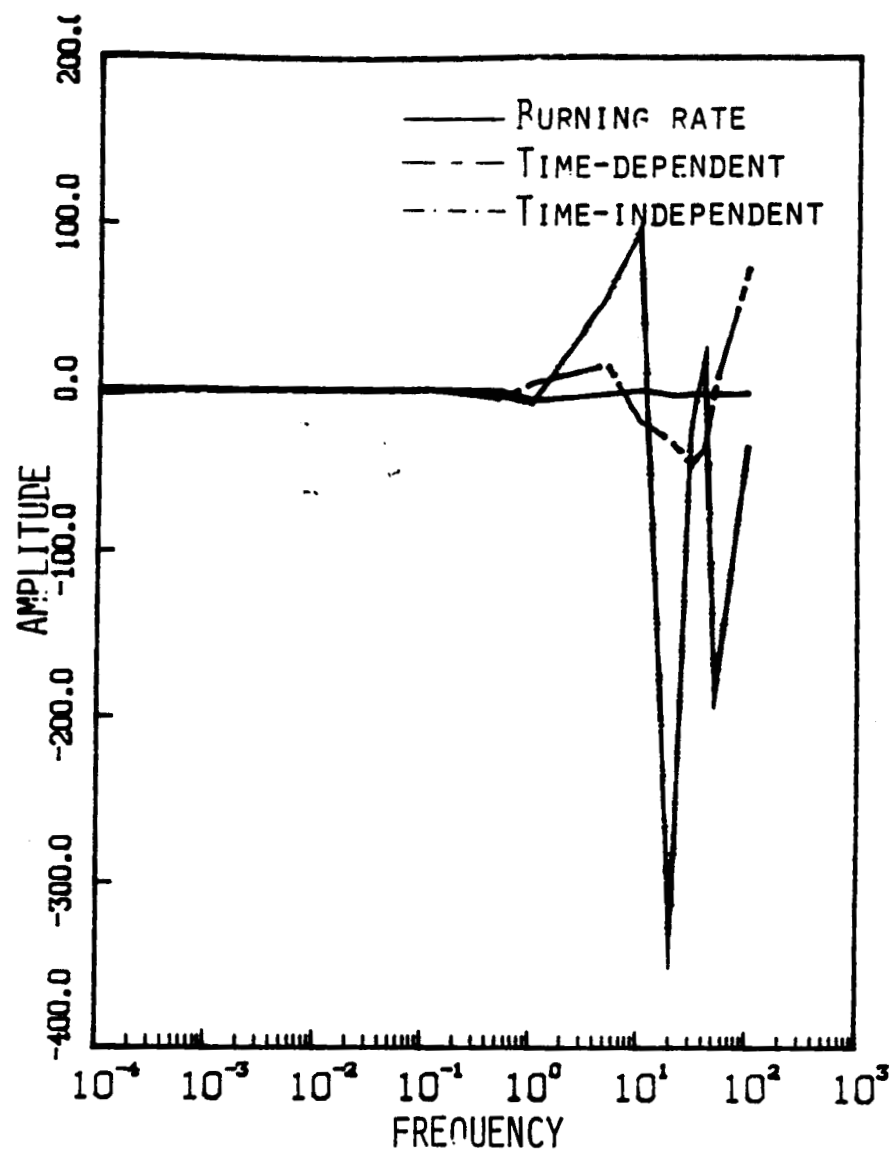


Fig. 28 Offset burning rate and acoustic admittances of the second order time-dependent and time-independent systems. Negative value of the burning rate at the frequency region verifies the past investigations.

APPENDIX F
COMPUTER PROGRAM LISTING

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C  SPCI.3
C  THIS IS SPCI(SOLID PROPELLANT COMBUSTION INSTABILITY)
C  PROGRAM TO SOLVE THE COMBUSTION INSTABILITY SUBJECT
C  TO ACOUSTIC PRESSURE DISTURBANCE IN THE SOLID PRO-
C  PELLANTS.
C*****
C*  TO SOLVE THE 1ST AND 2ND PERTURBATION SYSTEM ON  *
C*  2-DIM.NON-LIN.TIME-DEPEND.COMBUSTION OF A SOLID  *
C*  ROCKET PROPELLANTS FOR FINDING  *
C*  OUT NATURAL FREQUENCY AND RESPOND FUNCTION  *
C*****
C
C  A: TOTAL MATRIX (COMPLEX)
C  JC,V: FOR MATH PACK SUBROUTINE CGJR
C  CGJR: CALCULATION OF TOTAL MATRIX EQN (MATH-PACK)
C
C  ITYPE -1 : EIGENVALUE PROBLEM
C  ITYPE -* : INSTABILITY CALCULATION
C  IORDER-1 : FIRST ORDER SYSTEM ONLY
C  IORDER-* : SECOND ORDER CALCULATION
C  ITIME -1 : 2ND ORDER TIME INDEPENDENT
C  ITIME -* : 2ND ORDER TIME DEPENDENT
C  ITRT : NUMBER OF CALCULATION W.R.T. FREQUENCY
C
C
C  VIRTUAL  A
C
C  PARAMETER NW-5,NL-11,L4-4,NV-6
C  PARAMETER ND=NW*NL,NE=(NW-1)*(NL-1),NB=NW*4
C  PARAMETER NN=ND*NV,NQ=NN+NB,MC=NQ+1
C
C  IMPLICIT DOUBLE PRECISION (A-H,O-Z)
C  COMPLEX A(NQ,MC),V
C  COMPLEX FST(NN)
C
C  INTEGER JC(NQ)
C
C  COMMON/FIRST/FST
C
C  V=CMPLX(4.0,0.0)
C  ITYPE = 2
C  IORDER= 2
C  ITIME = 1
C  OME = 1.0
C
C  CALL GEOMET(WD,HT)
C  CALL DATAIN
C  CALL STEADY
C  CALL ASSM1(A,ITYPE,OME)
C
C  IF (ITYPE.EQ.1) THEN

```

```

      CALL EIGEN(A)
      ELSE
      CALL BNDRY1(A,OME,HT)
      ENDIF
C
C-----SOLUTION BY CGJR SUBROUTINE
C
      CALL CGJR(A,MC,NQ,NQ,MC,$801,JC,V)
C
C----- RESERVE 1ST ORDER SOLUTION
C
      DO 100 I=1,NN
      FST(I)=A(I,MC)
      WRITE(10,*) FST(I)
100 CONTINUE
C
      CALL OUTPUT(1,FST)
C
      IF(IORDER.EQ.1) GO TO 5000
      CALL ASSM2(A,ITIME)
      CALL BNDRY2(A,OME,ITIME,HT)
      CALL CGJR(A,MC,NQ,NQ,MC,$801,JC,V)
C
C----- RESERVE 2ND ORDER SOLUTION
C
      DO 200 I=1,NN
      FST(I)=A(I,MC)
      WRITE(20,*) FST(I)
200 CONTINUE
C
      CALL OUTPUT(2,FST)
C
      PRINT*,JC(2)
801 PRINT*, '++ERROR(1ST OR 2ND ORDER) ++',JC(1)
C
5000 CONTINUE
C
      STOP
      END
C

```

```
C      SUBROUTINE GEOMET(WD,HT)
C      *****
C      SUBROUTINE GEOMET (PREPARATION FOR CALCULATION DOMAIN)
C      *****
C
C      PARAMETER NW=5,NL=11,L4=4
C      PARAMETER ND=NW*NL,NE=(NW-1)*(NL-1)
C
C      IMPLICIT DOUBLE PRECISION (A-H,O-Z)
C      COMMON/DIMEN/ IE(NE,L4),X(ND),Y(ND)
C
C      WIDE=6.0D+0
C      HITE=3.0D+0
C
C      C----- (1) DEFINE GLOBAL NODES
C
C      DO 10 I=1,NE
C      IE(I,1)=(I-1)/(NL-1)+I+1
C      IE(I,2)=IE(I,1)+NL
C      IE(I,3)=IE(I,2)-1
C      IE(I,4)=IE(I,1)-1
C 10 CONTINUE
C
C      C----- (2) DEFINE GLOBAL COORD. X AND Y
C
C      WD=WIDE/(NW-1)
C      HT=HITE/(NL-1)
C      K=0
C      DO 20 I=1,NW
C      DO 20 J=1,NL
C      K=K+1
C      X(K)=(I-1)*WD
C      Y(K)=HITE-(J-1)*HT
C      IF ( Y(K).LE.0.001) THEN
C          Y(K)=0.0
C      ENDIF
C 20 CONTINUE
C
C      WRITE(6,900)
C 900 FORMAT(1H1,1X,'GLOBAL NODE, COORD., INPUT DATA'//)
C
C      DO 500 I=1,NE
C 500 WRITE(6,910) I,(IE(I,K),K=1,L4)
C 910 FORMAT(1H ,3X,'NE=',I3,5X,4I5)
C      PRINT*, 'X-',(X(I),I=1,ND-NL+1,NL)
C      PRINT*, 'Y-',(Y(I),I=1,NL)
C
C      RETURN
C      END
```

SUBROUTINE DATAIN

```

C
C *****
C SUBROUTINE DATAIN( INPUT DATA )
C *****
C     TC - TEMPERATURE AT COLD SIDE PROPELLANT
C     TS - TEMPERATURE AT SURFACE
C     HO - HEAT OF COMBUSTION PER UNIT MASS
C     EO - GAS PHASE ACTIVATION ENERGY
C     ES - SURFACE ACTIVATION ENERGY
C     QL - LATENT HEAT OF VAPORIZATION
C     DEL1 -  $KS \cdot CP / K \cdot CS$ 
C     DEL2 -  $K / KS$ 
C     BE - DENSITY RATIO ( $RS / RO$ )
C     GAM - SPECIFIC RATIO
C     PR - PRANDTL NUMBER
C     BM - MACH NUMBER
C
C
C     IMPLICIT DOUBLE PRECISION (A-H,O-Z)
C     COMMON/GIVEN/TC,TS,HO,EO,ES,QL,DEL1,
C     &          DEL2,BE,GAM,PR,BM
C
C     TC=0.15D+0
C     TS=0.35D+0
C     HO=1.30D+0
C     EO=1.0D+01
C     ES=4.00D+0
C     QL=0.15D+0
C     DEL1=1.00D+0
C     DEL2=1.00D+0
C     BE=1.0D+03
C     GAM=1.2D+00
C     PR=1.0D+00
C     BM=0.3D-02
C
C     WRITE(6,914)
C     PRINT*, 'TS, TC, HO, ES, EO, QL, DEL1, DEL2, GAM, BE, PR, BM'
C     PRINT*,  TS,TC,HO,ES,EO,QL,DEL1,DEL2,GAM,BE,PR,BM
914  FORMAT(//,1X,'INPUT CONSTANTS:')
C
C     RETURN
C     END
C

```

[illegible]

```

C      WZG(I)=2565000.0*((1.0-TZG(I))/TZG(I))**2
C      WZG(I)=2848208.85*((1.0-TZG(I))/TZG(I))**2
      &      *EXP(-EO/TZG(I))
200  CONTINUE
C
      WRITE(6,900)
      WRITE(6,910)
      DO 500 I=1,ND
      WRITE(6,920) I,RZG(I),UZG(I),VZG(I),TZG(I),SZG(I),PZG(I),WZG(I)
500  CONTINUE
900  FORMAT(1H1, 5X,'ZEROTH ORDER(STEADY) SOLUTION'//)
910  FORMAT(5X,'NODE',6X,'RO',11X,'U',12X,'V',
      &      12X,'T',12X,'Y',12X,'P',12X,'W')
920  FORMAT(5X,I5,7(E10.3,3X))
C
      RETURN
      END
C

```



```

SUBROUTINE ASSM1(A, ITYPE, OME)
C
C*****
C      SUBROUTINE ASSM1
C*****
C      INTPOL : INTERPOLATION AND ITS DERIVATIVES
C      CGJR  : CALCULATION OF TOTAL MATRIX EQN (MATH-PACK)
C
C      PARAMETER NW=5, NL=11, L4=4, NV=6
C      PARAMETER ND=NW*NL, NE=(NW-1)*(NL-1), NB=NW*4
C      PARAMETER NN=ND*NV, NQ=NN+NB, MC=NQ+1
C
C      IMPLICIT DOUBLE PRECISION (A-H,O-Z)
C      COMPLEX A(NQ,MC)
C
C      COMMON/GRAD/ PNNL(L4,L4), PKYL(L4,L4), PNXL(L4,L4),
&                PNYL(L4,L4), PDDL(L4,L4),
&                PXXL(L4,L4), PYYL(L4,L4)
C      COMMON/LOCAL1/AC2(L4,L4), AC3(L4,L4), AC4(L4,L4),
&                AC5(L4,L4), AC6(L4,L4), AC7(L4,L4),
&                AMX1(L4,L4), AMX2(L4,L4), AMX3(L4,L4),
&                AMX4(L4,L4), AMX5(L4,L4), AMX6(L4,L4),
&                AMX9(L4,L4), AMX11(L4,L4), AMX12(L4,L4),
&                AMY1(L4,L4), AMY2(L4,L4), AMY3(L4,L4),
&                AMY4(L4,L4), AMY5(L4,L4), AMY6(L4,L4),
&                AMY9(L4,L4), AMY11(L4,L4), AMY12(L4,L4),
&                AE1(L4,L4), AE2(L4,L4), AE3(L4,L4),
&                AE4(L4,L4), AE5(L4,L4), AE6(L4,L4),
&                AE7(L4,L4), AE8(L4,L4), AE9(L4,L4),
&                AE10(L4,L4), AE11(L4,L4), AE12(L4,L4),
&                AE14(L4,L4), AE15(L4,L4),
&                ASP1(L4,L4), ASP2(L4,L4), ASP3(L4,L4),
&                ASP4(L4,L4), ASP5(L4,L4), ASP6(L4,L4),
&                ASP7(L4,L4), ASP8(L4,L4), ASP9(L4,L4),
&                APRS1(L4,L4)
C      COMMON/DIMEN/IE(NE,L4), X(ND), Y(ND)
C      COMMON/ZERO/RZG(ND), UZG(ND), VZG(ND), TZG(ND),
&                SZG(ND), PZG(ND), WZG(ND)
C      COMMON/GIVEN/TC, TS, HO, EO, ES, QL, DEL1,
&                DEL2, BE, GAM, PR, BM
C      COMMON/COEFF/C1Z(ND,ND), C2Z(ND,ND), C3Z(ND,ND), C4Z(ND,ND),
&                X1Z(ND,ND), X2Z(ND,ND), X3Z(ND,ND), X4Z(ND,ND),
&                X5Z(ND,ND),
&                Y1Z(ND,ND), Y2Z(ND,ND), Y3Z(ND,ND), Y4Z(ND,ND),
&                Y5Z(ND,ND),
&                E1Z(ND,ND), E2Z(ND,ND), E3Z(ND,ND), E4Z(ND,ND),
&                E5Z(ND,ND), E6Z(ND,ND), E7Z(ND,ND),
&                S1Z(ND,ND), S2Z(ND,ND), S3Z(ND,ND), S4Z(ND,ND),
&                S5Z(ND,ND), S6Z(ND,ND), PRES1(ND,ND),
&                ANXZ(ND,ND), ANYZ(ND,ND)
C
C

```

```

      GMB2=1.0/(GAM*BM**2)
C*   PR3=1.0/3.0*PR
      PR3=0.0
C
C----- (1) INITIALIZATION
C
      DO 10 J=1,ND
      DO 10 I=1,ND
      ANXZ(I,J)=0.0
      ANYZ(I,J)=0.0
C
      C1Z(I,J)=0.0
      C2Z(I,J)=0.0
      C3Z(I,J)=0.0
      C4Z(I,J)=0.0
      X1Z(I,J)=0.0
      X2Z(I,J)=0.0
      X3Z(I,J)=0.0
      X4Z(I,J)=0.0
      X5Z(I,J)=0.0
      Y1Z(I,J)=0.0
      Y2Z(I,J)=0.0
      Y3Z(I,J)=0.0
      Y4Z(I,J)=0.0
      Y5Z(I,J)=0.0
      E1Z(I,J)=0.0
      E2Z(I,J)=0.0
      E3Z(I,J)=0.0
      E4Z(I,J)=0.0
      E5Z(I,J)=0.0
      E6Z(I,J)=0.0
      E7Z(I,J)=0.0
      S1Z(I,J)=0.0
      S2Z(I,J)=0.0
      S3Z(I,J)=0.0
      S4Z(I,J)=0.0
      S5Z(I,J)=0.0
      S6Z(I,J)=0.0
      PRES1(I,J)=0.0
10   CONTINUE
C
C----- (2) GLOBAL SUMMATION FOR EACH ELEMENT
C
      DO 700 LE=1,NE
C
C----- INTERPOLATION FUNCION AND ITS DERIVATIVES
C
      CALL INTPOL(LE,1,ITIME)
C
C
      DO 500 M=1,L4
      J=IE(LE,M)
      DO 500 N=1,L4

```

```

      I=IE(LE,N)
C
C----- (3) BLOCK ASSM:TERM BY TERM GLOBAL ASSME
C
      ANXZ(I,J)=ANXZ(I,J)+PNXL(N,M)
      ANYZ(I,J)=ANYZ(I,J)+PNYL(N,M)
C
      C1Z(I,J)= C1Z(I,J)+PNNL(N,M)
      C2Z(I,J)= C2Z(I,J)+ AC2(N,M)+ AC3(N,M)
      C3Z(I,J)= C3Z(I,J)+ AC4(N,M)+ AC5(N,M)
      C4Z(I,J)= C4Z(I,J)+ AC6(N,M)+ AC7(N,M)
C
      X1Z(I,J)= X1Z(I,J)+AMX1(N,M)
C      &      +GMB2*(AMX2(N,M)+AMX3(N,M))
      X2Z(I,J)= X2Z(I,J)+AMX4(N,M)
      X3Z(I,J)= X3Z(I,J)+AMX5(N,M)+AMX6(N,M)
C      &      +PR*PDDL(N,M)+PR3*PXXL(N,M)
      X4Z(I,J)= X4Z(I,J)+AMX9(N,M)+PR3*PXYL(N,M)
      X5Z(I,J)= X5Z(I,J)+AMX11(N,M)+AMX12(N,M)
C
      Y1Z(I,J)= Y1Z(I,J)+AMY1(N,M)
C      &      +GMB2*(AMY2(N,M)+AMY3(N,M))
      Y2Z(I,J)= Y2Z(I,J)+AMY4(N,M)
      Y3Z(I,J)= Y3Z(I,J)+AMY5(N,M)+AMY9(N,M)
C      &      +PR*PDDL(N,M)+PR3*PYYL(N,M)
      Y4Z(I,J)= Y4Z(I,J)+AMY6(N,M)+PR3*PXYL(N,M)
      Y5Z(I,J)= Y5Z(I,J)+AMY11(N,M)+AMY12(N,M)
C
      E1Z(I,J)= E1Z(I,J)+ AE1(N,M)
C      &      -2.0*HO*AE4(N,M)
C      &      +RGAM*(AE2(N,M)+ AE3(N,M)) -2.0*HO*AE4(N,M)
      E2Z(I,J)= E2Z(I,J)+AE5(N,M)
C      &      +RGAM*(AE6(N,M)+AE7(N,M))
      E3Z(I,J)= E3Z(I,J)+AE8(N,M)
C      &      +RGAM*(AE9(N,M)+AE10(N,M))
      E4Z(I,J)= E4Z(I,J)+AE11(N,M)
      E5Z(I,J)= E5Z(I,J)+AE12(N,M)
C      &      +PDDL(N,M)-HO*EO*AE14(N,M)
      E6Z(I,J)= E6Z(I,J)+AE15(N,M)
      E7Z(I,J)= E7Z(I,J)+ AE8(N,M)
C
      S1Z(I,J)= S1Z(I,J)+ASP1(N,M)+2.0*ASP2(N,M)
      S2Z(I,J)= S2Z(I,J)+ASP3(N,M)
      S3Z(I,J)= S3Z(I,J)+ASP4(N,M)
      S4Z(I,J)= S4Z(I,J)+ASP5(N,M)
      S5Z(I,J)= S5Z(I,J)+ASP6(N,M)
      S6Z(I,J)= S6Z(I,J)+ASP7(N,M)
C      &      +PDDL(N,M)+ASP9(N,M)*2.0
C
      PRES1(I,J)= PRES1(I,J)+APRS1(N,M)
500 CONTINUE
700 CONTINUE
C

```

C----- (4) CONSTRUCTION OF GLOBAL MATRIX

C

CALL GMATRX(A,OME,ITYPE)

C

RETURN

END

C

```

SUBROUTINE GMATRX(A,OME,ITYPE)
C
C*****
C      SUBROUTINE GMATRX( GLOBAL MATRIX BEFORE APPLYING
C                          BOUNDARY CONDITIONS )
C*****
C
C
C      PARAMETER NW=5,NL=11,L4=4,NV=6
C      PARAMETER ND=NW*NL,NE=(NW-1)*(NL-1),NB=NW*4
C      PARAMETER NN=ND*NV,NQ=NN+NB,MC=NQ+1
C
C      IMPLICIT DOUBLE PRECISION (A-H,O-Z)
C      COMPLEX A(NQ,MC)
C
C      COMMON/COEFF/ C1Z(ND,ND),C2Z(ND,ND),C3Z(ND,ND),C4Z(ND,ND),
&      X1Z(ND,ND),X2Z(ND,ND),X3Z(ND,ND),X4Z(ND,ND),
&      X5Z(ND,ND),
&      Y1Z(ND,ND),Y2Z(ND,ND),Y3Z(ND,ND),Y4Z(ND,ND),
&      Y5Z(ND,ND),
&      E1Z(ND,ND),E2Z(ND,ND),E3Z(ND,ND),E4Z(ND,ND),
&      E5Z(ND,ND),E6Z(ND,ND),E7Z(ND,ND),
&      S1Z(ND,ND),S2Z(ND,ND),S3Z(ND,ND),S4Z(ND,ND),
&      S5Z(ND,ND),S6Z(ND,ND),PRES1(ND,ND),
&      ANXZ(ND,ND),ANYZ(ND,ND)
C      COMMON/DIMEN/IE(NE,L4),X(ND),Y(ND)
C      COMMON/ZERO/RZG(ND),UZG(ND),VZG(ND),TZG(ND),
&      SZG(ND),PZG(ND),WZG(ND)
C      COMMON/GIVEN/TC,TS,HO,EO,ES,QL,DEL1,
&      DEL2,BE,GAM,PR,BM
C
C
C----- (1) BLOCK RI : CONSTRUCTION OF GLOBAL MATRIX
C
C      ND2=2*ND
C      ND3=3*ND
C      ND4=4*ND
C      ND5=5*ND
C
C      RGAM=(GAM-1.0)/GAM
C      GMB2=1.0/(GAM*BM**2)
C
C      DO 1000 J=1,ND
C      M1=J+ND
C      M2=J+ND2
C      M3=J+ND3
C      M4=J+ND4
C      M5=J+ND5
C      DO 1000 I=1,ND
C      N1=I+ND
C      N2=I+ND2
C      N3=I+ND3

```

N4-I+ND4

N5-I+ND5

C

IF(ITYPE.EQ.1) THEN

CRZI- C1Z(I,J)

XUZI- X2Z(I,J)

YVZI- Y2Z(I,J)

ETZI- E4Z(I,J)

EPZI--C1Z(I,J)*RGAM

ELSE

CRZI- C1Z(I,J)*OME

XUZI- X2Z(I,J)*OME

YVZI- Y2Z(I,J)*OME

ETZI- E4Z(I,J)*OME

EPZI--C1Z(I,J)*OME*RGAM

SSZI- S5Z(I,J)*OME

ENDIF

C

C CRZI- C1Z(I,J)*OME

CRZR- C2Z(I,J)

CUZR- C3Z(I,J)

CVZR- C4Z(I,J)

C

XRZR- X1Z(I,J)

C

XUZI- X2Z(I,J)*OME

XUZR- X3Z(I,J)

XVZR- X4Z(I,J)

XPZR-ANXZ(I,J)*GMB2

C

YRZR- Y1Z(I,J)

C

YVZI- Y2Z(I,J)*OME

YVZR- Y3Z(I,J)

YUZR- Y4Z(I,J)

YPZR-ANYZ(I,J)*GMB2

C

ERZR- E1Z(I,J)

EUZR- E2Z(I,J)

EVZR- E3Z(I,J)

C

ETZI- E4Z(I,J)*OME

ETZR- E5Z(I,J)

ESZR--E6Z(I,J)*2.0*HO

C

EPZI--C1Z(I,J)*RGAM*OME

C

SRZR- S1Z(I,J)

SUZR- S2Z(I,J)

SVZR- S3Z(I,J)

STZR- S4Z(I,J)*EO

C

SSZI- S5Z(I,J)*OME

SSZR- S6Z(I,J)

C

CC PRZR2-PRES1(I,J)

CC PTZR2- E4Z(I,J)

CC PPZR2- -C1Z(I,J)

C

```

IF(I.EQ.J) THEN
PRZR2= TZG(I)
PTZR2= RZG(I)
PPZR2= -1.00
ELSE
PRZR2=0.000
PTZR2=0.000
PPZR2=0.000
ENDIF

```

C

C----- (2) BLOCK COEFF

C

```

A(I,J) -CMPLX(CRZR,CRZI)
A(I,M1) -CMPLX(CUZR,0.0)
A(I,M2) -CMPLX(CVZR,0.0)
A(I,M3) -CMPLX(0.0,0.0)
A(I,M4) -CMPLX(0.0,0.0)
A(I,M5) -CMPLX(0.0,0.0)

```

C

```

A(N1,J) -CMPLX(XRZR,0.0)
A(N1,M1) -CMPLX(XUZR,XUZI)
A(N1,M2) -CMPLX(XVZR,0.0)
A(N1,M3) -CMPLX(0.00,0.0)
A(N1,M4) -CMPLX(0.0,0.0)
A(N1,M5) -CMPLX(XPZR,0.0)

```

C

```

A(N2,J) -CMPLX(YRZR,0.0)
A(N2,M1) -CMPLX(YUZR,0.0)
A(N2,M2) -CMPLX(YVZR,YVZI)
A(N2,M3) -CMPLX(0.00,0.0)
A(N2,M4) -CMPLX(0.0,0.0)
A(N2,M5) -CMPLX(YPZR,0.0)

```

C

```

A(N3,J) -CMPLX(ERZR,0.0)
A(N3,M1) -CMPLX(EUZR,0.0)
A(N3,M2) -CMPLX(EVZR,0.0)
A(N3,M3) -CMPLX(ETZR,ETZI)
A(N3,M4) -CMPLX(ESZR,0.0)
A(N3,M5) -CMPLX(0.00,EPZI)

```

C

```

A(N4,J) -CMPLX(SRZR,0.0)
A(N4,M1) -CMPLX(SUZR,0.0)
A(N4,M2) -CMPLX(SVZR,0.0)
A(N4,M3) -CMPLX(STZR,0.0)
A(N4,M4) -CMPLX(SSZR,SSZI)
A(N4,M5) -CMPLX(0.00,0.0)

```

C

```

A(N5,J) -CMPLX(PRZR2, 0.0)
A(N5,M1) -CMPLX( 0.0, 0.0)
A(N5,M2) -CMPLX( 0.0, 0.0)
A(N5,M3) -CMPLX(PTZR2, 0.0)
A(N5,M4) -CMPLX( 0.0, 0.0)

```

```

      A(N5,M5)=CMPLX(PPZR2, 0.0)
C
1000 CONTINUE
C
C----- (3) BLOCK SPACE : PREPERATION FOR LAGRANGE MULTIPLIER
C              APPLICATION
C----- ZERO SETS FOR TOTAL MATRIX
C
      DO 100 J=1,NQ
      DO 110 I=NN+1,NQ
      A(I,J)=CMPLX(0.0,0.0)
110 CONTINUE
      A(J,MC)=CMPLX(0.0,0.0)
100 CONTINUE
C
C----- (4) BLOCK RSV : RESERVATION FOR HIGHER ORDER CALCULATION
C
      DO 2000 J=1,MC
      DO 2000 I=1,NQ
      WRITE(7,7777) A(I,J)
2000 CONTINUE
7777 FORMAT(5X,E27.18,E27.18)
C
      RETURN
      END
C

```



```

      SUBROUTINE ASSM2(A,ITIME)
C
C*****
C      SUBROUTINE ASSM2
C*****
C
C
      PARAMETER NW=5,NL=11,L4=4,NV=6
      PARAMETER ND=NW*NL,NE=(NW-1)*(NL-1),NB=NW*4
      PARAMETER NN=ND*NV,NQ=NN+NB,MC=NQ+1
C
      IMPLICIT DOUBLE PRECISION (A-H,O-Z)
      COMPLEX A(NQ,MC),FST(NN)
      COMPLEX CF,XF,YF,EF,SF,PF,
&          GC,GX,GY,GE,GS,GP,
&          ACG,AXG,AYG,AEG,ASG,APG
C
      COMMON/LOCAL2/ ACG(L4),AXG(L4),AYG(L4),
&          AEG(L4),ASG(L4),APG(L4)
      COMMON/DIMEN/IE(NE,L4),X(ND),Y(ND)
      COMMON/FIRST/FST
C
      DIMENSION CF(ND),XF(ND),YF(ND),EF(ND),SF(ND),PF(ND)
C
      CLOSE(7)
      OPEN(7)
      DO 2000 J=1,MC
      DO 2000 I=1,NQ
          READ(7,7777) A(I,J)
2000 CONTINUE
7777 FORMAT(5X,E27.18,E27.18)
C
      ND2=2*ND
      ND3=3*ND
      ND4=4*ND
      ND5=5*ND
C
C----- (1) INITIALIZATION
C
      DO 100 J=1,ND
C
      CF(J)=CMPLX(0.0,0.0)
      XF(J)=CMPLX(0.0,0.0)
      YF(J)=CMPLX(0.0,0.0)
      EF(J)=CMPLX(0.0,0.0)
      SF(J)=CMPLX(0.0,0.0)
      PF(J)=CMPLX(0.0,0.0)
100 CONTINUE
C
      DO 700 LE=1,NE
C

```

```

C
C----- (2) INTEGRATION BY GAUSSIAN QUADRATURE
C
      CALL INTPOL(LE,2,ITIME)
C
C----- (3) BLOCK ASSME : GLOBAL ASSEMBLING OF SOURCE TERM
C
      DO 200 M=1,L4
      I=IE(LE,M)
C
      CF(I)=CF(I)+ACG(M)
      XF(I)=XF(I)+AXG(M)
      YF(I)=YF(I)+AYG(M)
      EF(I)=EF(I)+AEG(M)
      SF(I)=SF(I)+ASG(M)
      PF(I)=PF(I)+APG(M)
C
      200 CONTINUE
      700 CONTINUE
C
C----- (4) BLOCK COEFF: COEFFICIENT OF SOURCE TERM
C
      DO 300 J=1,ND
C
      M1=J+ND
      M2=J+ND2
      M3=J+ND3
      M4=J+ND4
      M5=J+ND5
C
      GC=-0.5*CF(J)
      GX=-0.5*XF(J)
      GY=-0.5*YF(J)
      GE=-0.5*EF(J)
      GS=-0.5*SF(J)
      GP=-0.5*PF(J)
C
      A(J,MC)= GC
      A(M1,MC)= GX
      A(M2,MC)= GY
      A(M3,MC)= GE
      A(M4,MC)= GS
      A(M5,MC)= GP
C*
      A(M5,MC)=-TF(J)*RF(J)*0.5
      300 CONTINUE
C
C----- (5) BLOCK RSV:RESERVATION FOR NEXT CALCULATION
C
      DO 400 J=1,ND
      M1=J+ND
      M2=J+ND2
      M3=J+ND3
      M4=J+ND4

```

```

M5=J+ND5
DO 400 I=1,ND
N1=I+ND
N2=I+ND2
N3=I+ND3
N4=I+ND4
N5=I+ND5

C
C----- (5-A) SUB-BLOCK RSV1:CHANGE OF COEFF. FOR 2ND TIME INDEPENDENT
C
      IF(ITIME.EQ.1) THEN
        A(I ,J )=CMPLX(REAL(A(I ,J )),0.0 )
        A(N1,M1)=CMPLX(REAL(A(N1,M1)),0.0 )
        A(N2,M2)=CMPLX(REAL(A(N2,M2)),0.0 )
        A(N3,M3)=CMPLX(REAL(A(N3,M3)),0.0 )
        A(N4,M4)=CMPLX(REAL(A(N4,M4)),0.0 )

C
C----- (5-B) SUB-BLOCK RSV2:CHANGE OF COEFF. FOR 2ND TIME DEPENDENT
C
      ELSE
        A(I ,J )=CMPLX(REAL(A(I ,J )),2.0*AIMAG(A(I ,J )))
        A(N1,M1)=CMPLX(REAL(A(N1,M1)),2.0*AIMAG(A(N1,M1)))
        A(N2,M2)=CMPLX(REAL(A(N2,M2)),2.0*AIMAG(A(N2,M2)))
        A(N3,M3)=CMPLX(REAL(A(N3,M3)),2.0*AIMAG(A(N3,M3)))
        A(N4,M4)=CMPLX(REAL(A(N4,M4)),2.0*AIMAG(A(N4,M4)))
      ENDIF

C
400 CONTINUE

C
      DO 3000 J=1,MC
      DO 3000 I=1,NQ
        WRITE(8,7777) A(I,J)
3000 CONTINUE

C
      RETURN
      END

C

```

```

SUBROUTINE INTPOL(LE, IORDER, ITIME)
C
C *****
C SUBROUTINE INTPOL (INTERPOLATION FUNCION)
C *****
C INTPL... INTERPOLATION AND ITS DERIVATIVES
C DJ: DETERMINANTS OF JACOBIAN MATRIX
C IG: NO. OF GAUSS.POINT
C
C
C PARAMETER NW=5, NL=11, L4=4, NV=6
C PARAMETER NE=(NW-1)*(NL-1), ND=NW*NL, NB=NW*4
C PARAMETER NN=ND*NV, IG=4
C
C IMPLICIT DOUBLE PRECISION (A-H,O-Z)
C
C COMMON/DJP/DJ(IG,IG),
C & PIWX(L4,IG,IG), PIWY(L4,IG,IG),
C & PIWN(L4,IG,IG), H(IG), W(IG)
C COMMON/GRAD/ PNNL(L4,L4), PXYL(L4,L4), PNXL(L4,L4),
C & PNYL(L4,L4), PDDL(L4,L4),
C & PXXL(L4,L4), PYYL(L4,L4)
C COMMON/DIMEN/IE(NE,L4), X(ND), Y(ND)
C COMMON/ZERO/RZG(ND), UZG(ND), VZG(ND), TZG(ND),
C & SZG(ND), PZG(ND), WZG(ND)
C COMMON/LZERO/RZ(L4), UZ(L4), VZ(L4), TZ(L4), SZ(L4), PZ(L4), WZ(L4)
C
C DIMENSION A4(4), B4(4), BN(L4), CN(L4)
C
C CALL GAUSS(IG,H,W)
C
C-----BLOCK LOCAL
C
C DO 100 I=1,L4
C J=IE(LE,I)
C RZ(I)=RZG(J)
C UZ(I)=UZG(J)
C VZ(I)=VZG(J)
C TZ(I)=TZG(J)
C SZ(I)=SZG(J)
C PZ(I)=PZG(J)
C WZ(I)=WZG(J)
100 CONTINUE
C
C-----BLOCK JACOB : JACOBIAN CALCULATION
C
C N1=IE(LE,1)
C N2=IE(LE,2)
C N3=IE(LE,3)
C N4=IE(LE,4)
C A4(1)=X(N1)+X(N2)+X(N3)+X(N4)

```

```

A4(2)=-X(N1)+X(N2)+X(N3)-X(N4)
A4(3)=-X(N1)-X(N2)+X(N3)+X(N4)
A4(4)=X(N1)-X(N2)+X(N3)-X(N4)
B4(1)=Y(N1)+Y(N2)+Y(N3)+Y(N4)
B4(2)=-Y(N1)+Y(N2)+Y(N3)-Y(N4)
B4(3)=-Y(N1)-Y(N2)+Y(N3)+Y(N4)
B4(4)=Y(N1)-Y(N2)+Y(N3)-Y(N4)

```

C

```

BN(1)=-1.0
BN(2)= 1.0
BN(3)= 1.0
BN(4)=-1.0
CN(1)=-1.0
CN(2)=-1.0
CN(3)= 1.0
CN(4)= 1.0

```

C

```

DO 200 N=1,L4 -
DO 200 M=1,L4
PNNW=0.0
PXYW=0.0
PNXW=0.0
PNYW=0.0
PXXW=0.0
PYYW=0.0

```

C

```

DO 300 I=1,IG
DO 300 J=1,IG
PXPW=0.25*(A4(3)+A4(4)*H(I))
PYPW=0.25*(B4(3)+B4(4)*H(I))
PXPZ=0.25*(A4(2)+A4(4)*H(J))
PYPZ=0.25*(B4(2)+B4(4)*H(J))

```

C

```

DJ(I,J)=-PXPW*PYPZ+PYPW*PXPZ

```

C

```

PHIX=0.25*(1.0+BN(M)*H(I))
PHIY=0.25*(1.0+CN(M)*H(J))
PPPZ=BN(M)*PHIY
PPPN=CN(M)*PHIX
PIWX(M,I,J)=( PYPW*PPPZ-PYPZ*PPPN)/DJ(I,J)
PIWY(M,I,J)=(-PXPW*PPPZ+PXPZ*PPPN)/DJ(I,J)

```

C

```

PIWN(M,I,J)=4.0*PHIX*PHIY

```

C

```

WWDJ=W(I)*W(J)*DJ(I,J)

```

C

```

PNNW=PNNW+PIWN(N,I,J)*PIWN(M,I,J)*WWDJ
PXYW=PXYW+PIWX(N,I,J)*PIWY(M,I,J)*WWDJ
PNXW=PNXW+PIWN(N,I,J)*PIWX(M,I,J)*WWDJ
PNYW=PNYW+PIWN(N,I,J)*PIWY(M,I,J)*WWDJ
PXXW=PXXW+PIWX(N,I,J)*PIWX(M,I,J)*WWDJ
PYYW=PYYW+PIWY(N,I,J)*PIWY(M,I,J)*WWDJ

```

C

```

300  CONTINUE
      PNNL(N,M)-PNNW
      PKYL(N,M)-PKYW
      PNXL(N,M)-PNXW
      PNYL(N,M)-PNYW
      PXXL(N,M)-PXXW
      PYYL(N,M)-PYYW
      PDDL(N,M)-PXXW+PYYW
200  CONTINUE
C
      IF (IORDER.EQ.1) THEN
      CALL LASS1
      ELSE
      CALL LASS2(LE,OME,ITIME)
      ENDIF
C
      RETURN
      END
C

```

SUBROUTINE LASS1

SUBROUTINE LASS1 (LOCAL ASSEMBLING OF GENERAL TERM)

PARAMETER NW=5,NL=11,L4=4,NV=6
PARAMETER NE=(NW-1)*(NL-1),ND=NW*NL
PARAMETER IG=4,NN=ND*NV

IMPLICIT DOUBLE PRECISION (A-H,O-Z)
COMMON/DJP/DJ(IG,IG),
& PIWX(L4,IG,IG),PIWY(L4,IG,IG),
& PIWN(L4,IG,IG),H(IG),W(IG)
COMMON/GRAD/ PNNL(L4,L4), PXYL(L4,L4), PNXL(L4,L4),
& PNYL(L4,L4), PDDL(L4,L4),
& PXXL(L4,L4), PYYL(L4,L4)
COMMON/LOCAL1/AC2(L4,L4), AC3(L4,L4), AC4(L4,L4),
& AC5(L4,L4), AC6(L4,L4), AC7(L4,L4),
& AMX1(L4,L4), AMX2(L4,L4), AMX3(L4,L4),
& AMX4(L4,L4), AMX5(L4,L4), AMX6(L4,L4),
& AMX9(L4,L4),AMX11(L4,L4),AMX12(L4,L4),
& AMY1(L4,L4), AMY2(L4,L4), AMY3(L4,L4),
& AMY4(L4,L4), AMY5(L4,L4), AMY6(L4,L4),
& AMY9(L4,L4),AMY11(L4,L4),AMY12(L4,L4),
& AE1(L4,L4), AE2(L4,L4), AE3(L4,L4),
& AE4(L4,L4), AE5(L4,L4), AE6(L4,L4),
& AE7(L4,L4), AE8(L4,L4), AE9(L4,L4),
& AE10(L4,L4), AE11(L4,L4), AE12(L4,L4),
& AE14(L4,L4), AE15(L4,L4),
& ASP1(L4,L4), ASP2(L4,L4), ASP3(L4,L4),
& ASP4(L4,L4), ASP5(L4,L4), ASP6(L4,L4),
& ASP7(L4,L4), ASP8(L4,L4), ASP9(L4,L4),
& APRS1(L4,L4)
COMMON/LZERO/ RZ(L4),UZ(L4),VZ(L4),TZ(L4),
& SZ(L4),PZ(L4),WZ(L4)

DIMENSION ZRO(IG,IG),ZROX(IG,IG),ZROY(IG,IG),
& ZU(IG,IG), ZV(IG,IG), ZUV(IG,IG),
& ZTM(IG,IG),ZTMX(IG,IG),ZTMY(IG,IG),
& ZRU(IG,IG), ZRV(IG,IG),ZRUX(IG,IG),
& ZRUY(IG,IG),
& ZRVX(IG,IG),ZRVY(IG,IG),ZUU1(IG,IG),
& ZUU2(IG,IG),
& ZUT(IG,IG),ZRTX(IG,IG),ZRTY(IG,IG),
& ZWDT(IG,IG),ZWSP(IG,IG), ZWT(IG,IG),
& ZRSX(IG,IG),ZRSY(IG,IG),ZUVS(IG,IG),
& ZTUV(IG,IG),ZTRX(IG,IG),ZTRY(IG,IG),
& ZTRO(IG,IG), ZTU(IG,IG), ZTV(IG,IG)

C----- (1) BLOCK ZERO : ZEROth ORDER TERM EVALUATION

C

```
DO 100 I=1,IG
DO 100 J=1,IG
PNUZ=0.0
PXUZ=0.0
PYUZ=0.0
PNVZ=0.0
PXVZ=0.0
PYVZ=0.0
PNRZ=0.0
PXRZ=0.0
PYRZ=0.0
PNTZ=0.0
PXTZ=0.0
PYTZ=0.0
PXSZ=0.0
PYSZ=0.0
PWDI=0.0
PWSP=0.0
PWT =0.0
```

C

```
DO 110 M=1,L4
PN=PIWN(M,I,J)
PX=PIWX(M,I,J)
PY=PIWY(M,I,J)
```

C

```
PNUZ=PNUZ+PN*UZ(M)
C* PXUZ=PXUZ+PX*UZ(M)
PYUZ=PYUZ+PY*UZ(M)
PNVZ=PNVZ+PN*VZ(M)
C* PXVZ=PXVZ+PX*VZ(M)
PYVZ=PYVZ+PY*VZ(M)
PNRZ=PNRZ+PN*RZ(M)
C* PXRZ=PXRZ+PX*RZ(M)
PYRZ=PYRZ+PY*RZ(M)
PNTZ=PNTZ+PN*TZ(M)
C* PXTZ=PXTZ+PX*TZ(M)
PYTZ=PYTZ+PY*TZ(M)
C* PXSZ=PXSZ+PX*SZ(M)
PYSZ=PYSZ+PY*SZ(M)
PWDI=PWDI+PN*WZ(M)/TZ(M)**2
PWT =PWT +PN*WZ(M)*TZ(M)
IF (ABS(SZ(M)).LE.1.0E-04) THEN
  PWSP=PWSP
ELSE
  PWSP =PWSP +PN*WZ(M)/SZ(M)
ENDIF
```

C

110 CONTINUE

C

```
ZRO(I,J)=PNRZ
ZROX(I,J)=PXRZ
```



```

ZROY(I,J)-PYRZ
  ZU(I,J)-PNUZ
  ZV(I,J)-PNVZ
  ZUV(I,J)-PXUZ+PYVZ
  ZTM(I,J)-PNTZ
ZTMX(I,J)-PXTZ
ZTMY(I,J)-PYTZ
  ZRU(I,J)-PNRZ*PNUZ
  ZRV(I,J)-PNRZ*PNVZ
ZRUX(I,J)-PNRZ*PXUZ
ZRUY(I,J)-PNRZ*PYUZ
ZRVX(I,J)-PNRZ*PXVZ
ZRVY(I,J)-PNRZ*PYVZ
ZUU1(I,J)-PNUZ*PXUZ+PNVZ*PYUZ
ZUU2(I,J)-PNUZ*PXVZ+PNVZ*PYVZ
ZRTX(I,J)-PNRZ*PXTZ
ZRTY(I,J)-PNRZ*PYTZ
  ZUT(I,J)-PNUZ*PXTZ+PNVZ*PYTZ
ZWDI(I,J)-PWDI
ZWSP(I,J)-PWSP
  ZWT(I,J)-PWT
ZRSX(I,J)-PNRZ*PXSZ
ZRSY(I,J)-PNRZ*PYSZ
ZUVS(I,J)-PNUZ*PXSZ+PNVZ*PYSZ
  ZTU(I,J)-PNTZ*PNUZ
  ZTV(I,J)-PNTZ*PNVZ
ZTUV(I,J)-PNTZ*(PXUZ+PYVZ)
ZTRO(I,J)-1.0
C  ZTRO(I,J)-PNTZ*PNRZ
  ZTRX(I,J)-PNTZ*PXRZ
  ZTRY(I,J)-PNTZ*PYRZ
100 CONTINUE
C
C----- (2) BLOCK INTEG : GAUSSIAN INTEGRATION
C
DO 120 N=1,L4
DO 120 M=1,L4
  SC2=0.0
  SC3=0.0
  SC4=0.0
  SC5=0.0
  SC6=0.0
  SC7=0.0
  SMX1=0.0
  SMX2=0.0
  SMX3=0.0
  SMX4=0.0
  SMX5=0.0
  SMX6=0.0
  SMX9=0.0
  SMX11=0.0
  SMX12=0.0
  SMY1=0.0

```

SMY2=0.0
 SMY3=0.0
 SMY4=0.0
 SMY5=0.0
 SMY6=0.0
 SMY9=0.0
 SMY11=0.0
 SMY12=0.0
 SE1=0.0
 SE2=0.0
 SE3=0.0
 SE4=0.0
 SE5=0.0
 SE6=0.0
 SE7=0.0
 SE8=0.0
 SE9=0.0
 SE10=0.0
 SE11=0.0
 SE12=0.0
 SE14=0.0
 SE15=0.0
 SSP1=0.0
 SSP2=0.0
 SSP3=0.0
 SSP4=0.0
 SSP5=0.0
 SSP6=0.0
 SSP7=0.0
 SSP9=0.0
 SPRS1=0.0

C

DO 130 K=1, IG
 DO 130 L=1, IG
 WWDJ=W(K)*W(L)*DJ(K,L)
 PNN =PIWN(N,K,L)*PIWN(M,K,L)*WWDJ
 PNK =PIWN(N,K,L)*PIWX(M,K,L)*WWDJ
 PNY =PIWN(N,K,L)*PIWY(M,K,L)*WWDJ

C

SC2= SC2+(PNK *ZU(K,L)+PNY*ZV(K,L))
 SC3= SC3+PNN *ZUV(K,L)
 SC4= SC4+PNN*ZROX(K,L)
 SC5= SC5+PNK *ZRO(K,L)
 SC6= SC6+PNN*ZROY(K,L)
 SC7= SC7+PNY *ZRO(K,L)

C

SMX1=SMX1+PNN*ZUU1(K,L)
 SMX2=SMX2+PNK *ZTM(K,L)
 SMX3=SMX3+PNN*ZTMX(K,L)
 SMX4=SMX4+PNN *ZRO(K,L)
 SMX5=SMX5+(PNK*ZRU(K,L)+PNY*ZRV(K,L))
 SMX6=SMX6+PNN*ZRUX(K,L)
 SMX9=SMX9+PNN*ZRUY(K,L)

SMX11=SMX11+PNN*ZROX(K,L)
SMX12=SMX12+PNX *ZRO(K,L)

C

SMY1=SMY1+PNN*ZUU2(K,L)
SMY2=SMY2+PNY *ZTM(K,L)
SMY3=SMY3+PNN*ZTMY(K,L)
SMY4=SMY4+PNN *ZRO(K,L)
SMY5=SMY5+(PNX*ZRU(K,L)+PNY*ZRV(K,L))
SMY6=SMY6+PNN*ZRVX(K,L)
SMY9=SMY9+PNN*ZRVY(K,L)
SMY11=SMY11+PNN*ZROY(K,L)
SMY12=SMY12+PNY *ZRO(K,L)

C

SE1= SE1+PNN *ZUT(K,L)
SE2= SE2+PNX *ZTU(K,L)+PNY*ZTV(K,L)
SE3= SE3+PNN*ZTUV(K,L)
SE4= SE4+PNN* ZWT(K,L)
SE5= SE5+PNN*ZRTX(K,L)
SE6= SE6+PNN*ZTRX(K,L)
SE7= SE7+PNX*ZTRO(K,L)
SE8= SE8+PNN*ZRTY(K,L)
SE9= SE9+PNN*ZTRY(K,L)
SE10=SE10+PNY*ZTRO(K,L)
SE11=SE11+PNN *ZRO(K,L)
SE12=SE12+PNX *ZRU(K,L)+PNY*ZRV(K,L)
SE14=SE14+PNN*ZWDI(K,L)
SE15=SE15+PNN*ZWSP(K,L)

C

SSP1=SSP1+PNN*ZUVS(K,L)
SSP3=SSP3+PNN*ZRSX(K,L)
SSP4=SSP4+PNN*ZRSY(K,L)

C

SPRS1=SPRS1+PNN*ZTM(K,L)

C

130 CONTINUE

C

SSP2=SE4
SSP5=SE14
SSP6=SE11
SSP7=SE12
SSP9=SE15

C

C----- (3) BLOCK LTERM : LOCAL COEFFICIENT

C

AC2(N,M)= SC2
AC3(N,M)= SC3
AC4(N,M)= SC4
AC5(N,M)= SC5
AC6(N,M)= SC6
AC7(N,M)= SC7

C

AMX1(N,M)=SMX1
AMX2(N,M)=SMX2

AMX3(N,M)-SMX3
AMX4(N,M)-SMX4
AMX5(N,M)-SMX5
AMX6(N,M)-SMX6
AMX9(N,M)-SMX9
AMX11(N,M)-SMX11
AMX12(N,M)-SMX12

C

AMY1(N,M)-SMY1
AMY2(N,M)-SMY2
AMY3(N,M)-SMY3
AMY4(N,M)-SMY4
AMY5(N,M)-SMY5
AMY6(N,M)-SMY6
AMY9(N,M)-SMY9
AMY11(N,M)-SMY11
AMY12(N,M)-SMY12

C

AE1(N,M)- SE1
AE2(N,M)- SE2
AE3(N,M)- SE3
AE4(N,M)- SE4
AE5(N,M)- SE5
AE6(N,M)- SE6
AE7(N,M)- SE7
AE8(N,M)- SE8
AE9(N,M)- SE9
AE10(N,M)-SE10
AE11(N,M)-SE11
AE12(N,M)-SE12
AE14(N,M)-SE14
AE15(N,M)-SE15

C

ASP1(N,M)-SSP1
ASP2(N,M)-SSP2
ASP3(N,M)-SSP3
ASP4(N,M)-SSP4
ASP5(N,M)-SSP5
ASP6(N,M)-SSP6
ASP7(N,M)-SSP7
ASP9(N,M)-SSP9

C

APRS1(N,M)-SPRS1

C

120 CONTINUE

C

RETURN
END

C

SUBROUTINE LASS2(LE,OME,ITIME)

SUBROUTINE LASS2(LOCAL ASSEMBLING OF SOURCE TERM)

```

PARAMETER NW=5,NL=11,L4=4,NV=6
PARAMETER NE=(NW-1)*(NL-1),ND=NW*NL
PARAMETER IG=4,NN=ND*NV

```

IMPLICIT DOUBLE PRECISION (A-H,O-Z)

COMPLEX FST(NN)

COMPLEX RF,UF,VF,TF,SF,
& PNRF,PXRF,PYRF, PNUF,PXUF,PYUF,
& PNVF,PXVF,PYVF, PNTF,PXTF,PYTF,
& PNSF,PXSF,PYSF,
& FRO ,FRUV ,FRU ,FRV ,F011X,F011Y,
& F101X,F101Y,F110X,F110Y,FRT1X,FRT1Y,
& FRT2X,FRT2Y,FPTM ,FRVT1,FRVT2,FRVT3,
& FRUS1,FRUS2,FRUS3,FPRSP,FPWT2,FPWRT,
& FPWST,FPWRS,FPWR2,FPWS2,FTUVR,
& CG ,XG ,YG ,EG ,SG ,PG ,
& ACG,AXG,AYG,AEG,ASG,APG

```
COMMON/DJP/DJ(IG,IG),
&      PIWX(L4,IG,IG),PIWY(L4,IG,IG),
&      PIWN(L4,IG,IG),H(IG),W(IG)
COMMON/LOCAL2/ ACG(L4),AXG(L4),AYG(L4),
&      AEG(L4),ASG(L4),APG(L4)
COMMON/DIMEN/IE(NE,L4),X(ND),Y(ND)
COMMON/FIRST/FST
COMMON/GIVEN/TC,TS,HO,EO,ES,QL,DEL1,
&      DEL2,BE,GAM,PR,BM
```

```
COMMON/LZERO/ RZ(L4),UZ(L4),VZ(L4),TZ(L4),
&              SZ(L4),PZ(L4),WZ(L4)
DIMENSION      RF(L4),UF(L4),VF(L4),
&              TF(L4),SF(L4)
```

```

DIMENSION      FRO(IG,IG) , FRUV(IG,IG) ,
&              FRU(IG,IG) , FRV(IG,IG) ,
&              F011X(IG,IG) , F011Y(IG,IG) ,
&              F101X(IG,IG) , F101Y(IG,IG) ,
&              F110X(IG,IG) , F110Y(IG,IG) ,
&              FRT1X(IG,IG) , FRT1Y(IG,IG) ,
&              FRT2X(IG,IG) , FRT2Y(IG,IG) ,
&              FPTM(IG,IG) , FRVT1(IG,IG) ,
&              FRVT2(IG,IG) , FRVT3(IG,IG) ,
&              FRUS1(IG,IG) , FRUS2(IG,IG) ,
&              FRUS3(IG,IG) , FPRSP(IG,IG) ,
&              FPWT2(IG,IG) , FPWRT(IG,IG) ,

```

```

      &          FPWST(IG,IG),FPWRS(IG,IG),
      &          FPWR2(IG,IG),FPWS2(IG,IG),FTUVR(IG,IG)
C
C
C----- (1) BLOCK LOCAL : DEFINE LOCAL VARIABLES
C
      GMB2=0.0
      RGAM=(GAM-1.0)/GAM
      RGAM=0.0
      QGAM=1.0-2.0*RGAM
C
      DO 200 I=1,L4
          J=IE(LE,I)
C
      N1=J+ND
      N2=J+ND*2
      N3=J+ND*3
      N4=J+ND*4
      N5=J+ND*5
C
      RF(I)=FST(J)
      UF(I)=FST(N1)
      VF(I)=FST(N2)
      TF(I)=FST(N3)
      SF(I)=FST(N4)
C
      IF ( REAL(SF(I)).LE.1.0E-05) THEN
          SF(I)=0.0
          ENDIF
      200 CONTINUE
C
C----- (2) INITIALIZATION
C
      DO 210 I=1,IG
          DO 210 J=1,IG
C
      PNRZ=0.0
      PNUZ=0.0
      PXUZ=0.0
      PYUZ=0.0
      PNVZ=0.0
      PXVZ=0.0
      PYVZ=0.0
      PNTZ=0.0
      PXTZ=0.0
      PYTZ=0.0
      PNSZ=0.0
      PXSZ=0.0
      PYSZ=0.0
C
      PNUF=CMPLX(0.0,0.0)
      PXUF=CMPLX(0.0,0.0)
      PYUF=CMPLX(0.0,0.0)

```

PNVF-CMPLX(0.0,0.0)
 PXVF-CMPLX(0.0,0.0)
 PYVF-CMPLX(0.0,0.0)
 PNRF-CMPLX(0.0,0.0)
 PXRF-CMPLX(0.0,0.0)
 PYRF-CMPLX(0.0,0.0)
 PNTF-CMPLX(0.0,0.0)
 PXTF-CMPLX(0.0,0.0)
 PYTF-CMPLX(0.0,0.0)
 PNSF-CMPLX(0.0,0.0)
 PXSF-CMPLX(0.0,0.0)
 PYSF-CMPLX(0.0,0.0)

C

PWT2-0.0
 PWRT-0.0
 PWST-0.0
 PWR5-0.0
 PWR2-0.0
 PWS2-0.0

C

C----- (3) BLOCK SUM : SUMMATION

C

DO 220 M=1,L4
 PN=PIWN(M,I,J)
 PX=PIWX(M,I,J)
 PY=PIWY(M,I,J)

C

PNRZ=PNRZ+PN*RZ(M)
 PNUZ=PNUZ+PN*UZ(M)
 C* PXUZ=PXUZ+PX*UZ(M)
 PYUZ=PYUZ+PY*UZ(M)
 PNVZ=PNVZ+PN*VZ(M)

C*

PXVZ=PXVZ+PX*VZ(M)
 PYVZ=PYVZ+PY*VZ(M)

C*

PNTZ=PNTZ+PN*TZ(M)
 PXTZ=PXTZ+PX*TZ(M)
 PYTZ=PYTZ+PY*TZ(M)

C*

PXSZ=PXSZ+PX*SZ(M)
 PYSZ=PYSZ+PY*SZ(M)

C

PNUF=PNUF+PN*UF(M)
 PXUF=PXUF+PX*UF(M)
 PYUF=PYUF+PY*UF(M)
 PNVF=PNVF+PN*VF(M)
 PXVF=PXVF+PX*VF(M)
 PYVF=PYVF+PY*VF(M)
 PNRF=PNRF+PN*RF(M)
 PXRF=PXRF+PX*RF(M)
 PYRF=PYRF+PY*RF(M)
 PNTF=PNTF+PN*TF(M)
 PXTF=PXTF+PX*TF(M)
 PYTF=PYTF+PY*TF(M)
 PNSF=PNSF+PN*SF(M)

PXSF-PXSF+PX*SF(M)
PYSF-PYSF+PY*SF(M)

C

PWT2-PWT2+PN*(EO*WZ(M)*(0.5*EO/TZ(M)-1.0))/TZ(M)**3
PWRT-PWRT+PN*(2.0*EO*WZ(M)/TZ(M))
PWR2-PWR2+PN*WZ(M)/RZ(M)**2
IF (ABS(SZ(M)).LE.1.0E-04) THEN
PWST-PWST
PWRS-PWRS
PWS2-PWS2
ELSE
PWST-PWST+PN*(2.0*EO*WZ(M)/(SZ(M)*TZ(M)**2))
PWRS-PWRS+PN*(4.0*WZ(M)/(RZ(M)*SZ(M)))
PWS2-PWS2+PN*WZ(M)/SZ(M)**2
ENDIF

C

220 CONTINUE

CC

IF(ETIME.EQ.1) THEN

C

C----- (4) BLOCK TI : TIME INDEPENDENT SOURCE

FRO(I,J)=CONJG(PXRF)*PNUF+CONJG(PYRF)*PNVF
FRUV(I,J)=CONJG(PNRF)*(PXUF+PYVF)
FRU(I,J)=CONJG(PNRF)*PNUF
FRV(I,J)=CONJG(PNRF)*PNVF
F011X(I,J)=PNRZ*(CONJG(PNUF)*PXUF+CONJG(PNVF)*PYUF)
F011Y(I,J)=PNRZ*(CONJG(PNUF)*PXVF+CONJG(PNVF)*PYVF)
F101X(I,J)=CONJG(PNRF)*(PNUZ*PXUF+PNVZ*PYUF)
F101Y(I,J)=CONJG(PNRF)*(PNUZ*PXVF+PNVZ*PYVF)
F110X(I,J)=CONJG(PNRF)*(PNUF*PXUZ+PNVF*PYUZ)
F110Y(I,J)=CONJG(PNRF)*(PNUF*PXVZ+PNVF*PYVZ)
FRT1X(I,J)=CONJG(PXRF)*PNTF
FRT1Y(I,J)=CONJG(PYRF)*PNTF
FRT2X(I,J)=CONJG(PNRF)*PXTF
FRT2Y(I,J)=CONJG(PNRF)*PYTF
FPTM(I,J)=CONJG(PNRF)*PNTF
FRVT1(I,J)=PNRZ*(CONJG(PNUF)*PXTF+CONJG(PNVF)*PYTF)
FRVT2(I,J)=CONJG(PNRF)*(PNUZ*PXTF+PNVZ*PYTF)
FRVT3(I,J)=CONJG(PNRF)*(PNUF*PXTZ+PNVF*PYTZ)
FRUS1(I,J)=PNRZ*(CONJG(PNUF)*PXSF+CONJG(PNVF)*PYSF)
FRUS2(I,J)=CONJG(PNRF)*(PNUZ*PXSF+PNVZ*PYSF)
FRUS3(I,J)=CONJG(PNRF)*(PNUF*PXSZ+PNVF*PYSZ)
FPRSP(I,J)=CONJG(PNRF)*PNSF
FPWT2(I,J)=PWT2*CONJG(PNTF)*PNTF
FPWRT(I,J)=PWRT*CONJG(PNRF)*PNTF
FPWST(I,J)=PWST*CONJG(PNSF)*PNTF
FPWRS(I,J)=PWRS*CONJG(PNRF)*PNSF
FPWR2(I,J)=PWR2*CONJG(PNRF)*PNRF
FPWS2(I,J)=PWS2*CONJG(PNSF)*PNSF
FTUVR(I,J)=PNTZ*(CONJG(PXRF)*PNUF+CONJG(PYRF)*PNVF)
& +PNTZ*(CONJG(PNRF)*PXUF+CONJG(PNRF)*PYVF)

C

C

ELSE

C

C----- (5) BLOCK TD : TIME DEPENDENT SOURCE

CC

```
      FRO(I,J)=PXRF*PNUF+PYRF*PNVF
      FRUV(I,J)=PNRF*(PXUF+PYVF)
      FRU(I,J)=PNRF*PNUF
      FRV(I,J)=PNRF*PNVF
      F011X(I,J)=PNRZ*(PNUF*PXUF+PNVF*PYUF)
      F011Y(I,J)=PNRZ*(PNUF*PXVF+PNVF*PYVF)
      F101X(I,J)=PNRF*(PNUZ*PXUF+PNVZ*PYUF)
      F101Y(I,J)=PNRF*(PNUZ*PXVF+PNVZ*PYVF)
      F110X(I,J)=PNRF*(PNUF*PXUZ+PNVF*PYUZ)
      F110Y(I,J)=PNRF*(PNUF*PXVZ+PNVF*PYVZ)
      FRT1X(I,J)=PXRF*PNTF
      FRT1Y(I,J)=PYRF*PNTF
      FRT2X(I,J)=PNRF*PXTF
      FRT2Y(I,J)=PNRF*PYTF
      FPTM(I,J)=PNRF*PNTF
      FRVT1(I,J)=PNRZ*(PNUF*PXTF+PNVF*PYTF)
      FRVT2(I,J)=PNRF*(PNUZ*PXTF+PNVZ*PYTF)
      FRVT3(I,J)=PNRF*(PNUF*PXTZ+PNVF*PYTZ)
      FRUS1(I,J)=PNRZ*(PNUF*PXSZ+PNVF*PYSZ)
      FRUS2(I,J)=PNRF*(PNUZ*PXSZ+PNVZ*PYSZ)
      FRUS3(I,J)=PNRF*(PNUF*PXSZ+PNVF*PYSZ)
      FPRSP(I,J)=PNRF*PNSF
      FPWT2(I,J)=PWT2*PNTF*PNTF
      FPWRT(I,J)=PWRT*PNRF*PNTF
      FPWST(I,J)=PWST*PNSF*PNTF
      FPWRS(I,J)=PWSR*PNRF*PNSF
      FPWR2(I,J)=PWR2*PNRF*PNRF
      FPWS2(I,J)=PWS2*PNSF*PNSF
      FTUVR(I,J)=PNTZ*(PXRF*PNUF+PYRF*PNVF)
&      +PNTZ*(PNRF*PXUF+PNRF*PYVF)
```

C

ENDIF

C

210 CONTINUE

C

C----- (6) BLOCK LSOURCE

C

```
DO 230 N=1,L4
  CG=CMPLX(0.0,0.0)
  XG=CMPLX(0.0,0.0)
  YG=CMPLX(0.0,0.0)
  EG=CMPLX(0.0,0.0)
  SG=CMPLX(0.0,0.0)
  PG=CMPLX(0.0,0.0)
```

C

```
DO 240 I=1,IG
DO 240 J=1,IG
WWDJ=W(I)*W(J)*DJ(I,J)
  PN=PIWN(N,I,J)*WWDJ
```

C

```
CG=CG+PN*( FRO(I,J)+ FRUV(I,J))
XG=XG+PN*(F011X(I,J)+F101X(I,J)+F110X(I,J)+
& GMB2*(FRT1X(I,J)+FRT2X(I,J))+CMPLX(0.0,1.0)*OME*FRU(I,J))
YG=YG+PN*(F011Y(I,J)+F101Y(I,J)+F110Y(I,J)+
& GMB2*(FRT1Y(I,J)+FRT2Y(I,J))+CMPLX(0.0,1.0)*OME*FRV(I,J))
EG=EG+PN*(FRVT1(I,J)+FRVT2(I,J)+FRVT3(I,J)-
& HO*(FPWT2(I,J)+FPWRT(I,J)+FPWST(I,J)+
& FPWRS(I,J)+FPWR2(I,J)+FPWS2(I,J))+
& QGAM*CMPLX(0.0,1.0)*OME*FPTM(I,J)+ RGAM*FTUVR(I,J))
SG=SG+PN*(FRUS1(I,J)+FRUS2(I,J)+FRUS3(I,J)+
& FPWT2(I,J)+FPWRT(I,J)+FPWST(I,J)+
& FPWRS(I,J)+FPWR2(I,J)+FPWS2(I,J)+
& OME*CMPLX(0.0,1.0)* FPRSP(I,J))
PG=PG+PN*FPTM(I,J)
```

C

240 CONTINUE

C

```
ACG(N)-CG
AXG(N)-XG
AYG(N)-YG
AEG(N)-EG
ASG(N)-SG
APG(N)-PG
```

230 CONTINUE

C

```
RETURN
END
```

C

```

C      SUBROUTINE GAUSS(IG,H,W)
C      *****
C      SUBROUTINE GAUSS ( GAUSSIAN QUADRATURE INTEGRATION )
C      *****
C      IG: NO. OF GAUSS.POINT
C      H,W: ABSCISSAE AND WEIGHTING COEFF. OF GAUSS. POINTS
C
C      IMPLICIT DOUBLE PRECISION (A-H,O-Z)
C      DIMENSION H(IG), W(IG)
C
C      IF      (IG.EQ.1) THEN
C      H(1)=0.0D+00
C      W(1)=2.0D+00
C      ELSEIF(IG.EQ.2) THEN
C      H(1)=0.577350269189626D+00
C      H(2)=-H(1)
C      W(1)=1.0D+00
C      W(2)=1.0D+00
C      ELSEIF(IG.EQ.3) THEN
C      H(1)=0.774596669241483D+00
C      H(2)=0.0D+00
C      H(3)=-H(1)
C      W(1)=0.555555555555555D+00
C      W(2)=0.888888888888888D+00
C      W(3)=W(1)
C      ELSEIF(IG.EQ.4) THEN
C      H(1)=0.861136311594053D+00
C      H(2)=0.339981043584856D+00
C      H(3)=-H(2)
C      H(4)=-H(1)
C      W(1)=0.347854845137454D+00
C      W(2)=0.652145154862546D+00
C      W(3)=W(2)
C      W(4)=W(1)
C      ELSEIF(IG.EQ.5) THEN
C      H(1)=0.906179845938664D+00
C      H(2)=0.538469310105683D+00
C      H(3)=0.0D+00
C      H(4)=-H(2)
C      H(5)=-H(1)
C      W(1)=0.236926885056189D+00
C      W(2)=0.478628670499366D+00
C      W(3)=0.568888888888889D+00
C      W(4)=W(2)
C      W(5)=W(1)
C      ELSEIF(IG.EQ.6) THEN
C      H(1)=0.932469514203152D+00
C      H(2)=0.661209386466265D+00
C      H(3)=0.238619186083197D+00
C      H(4)=-H(3)
C      H(5)=-H(2)

```

H(6)--H(1)
W(1)-0.171324492379170D+00
W(2)-0.360761573048139D+00
W(3)-0.467913934572691D+00
W(4)-W(3)
W(5)-W(2)
W(6)-W(1)

C

ENDIF

C

RETURN
END

C

```

C      SUBROUTINE BNDY1 (A,OME,HT)
C
C      *****
C      SUBROUTINE BNDY1( BOUNDARY CONDITION APPLICATION )
C      *****
C
C
C      PARAMETER NW=5,NL=11,NV=6
C      PARAMETER ND=NW*NL,NE=(NW-1)*(NL-1),NB=NW*4
C      PARAMETER NN=ND*NV,NQ=NN+NB,MC=NQ+1
C
C      IMPLICIT DOUBLE PRECISION (A-H,O-Z)
C      COMPLEX A(NQ,MC),B(NB,NN),C(NN)
C      COMPLEX FST(NN)
C
C      COMMON/FIRST/FST
C
C
C      CALL BCEQN(B,OME,1,1,HT)
C
C      DO 100 J=1,NN
C      DO 100 I=1,NB
C          K=I+NN
C      A(K,J)=B(I,J)
C      A(J,K)=A(K,J)
100  CONTINUE
C
C      CALL DIRICH(A,1)
C      CALL NEUMN(C,1)
C
C      DO 110 I=1,NN
C      A(I,MC)=A(I,MC)+C(I)
110  CONTINUE
C
C      RETURN
C      END
C

```

```

C      SUBROUTINE BNDRY2 (A,OME,ITIME,HT)
C
C      *****
C      SUBROUTINE BNDRY2( BOUNDARY CONDITION APPLICATION )
C      *****
C
C      PARAMETER NW=5,NL=11,L4=4,NV=6
C      PARAMETER ND=NW*NL,NE=(NW-1)*(NL-1),NB=NW*4
C      PARAMETER NN=ND*NV,NQ=NN+NB,MC=NQ+1
C
C      IMPLICIT DOUBLE PRECISION (A-H,O-Z)
C      COMPLEX A(NQ,MC),B(NB,NN),C(NN)
C      COMPLEX FST(NN)
C
C      COMMON/FIRST/FST
C
C      CALL BCEQN(B,OME,2,ITIME,HT)
C
C      DO 200 I=1,NB
C      DO 200 J=1,NN
C          K=I+NN
C          A(K,J)=B(I,J)
C          A(J,K)=A(K,J)
200  CONTINUE
C
C      CALL DIRICH(A,2)
C      CALL NEUMN(C,2)
C
C      DO 210 I=1,NN
C          A(I,MC)=A(I,MC)+C(I)
210  CONTINUE
C
C      RETURN
C      END
C

```

```

SUBROUTINE BCEQN(B,OME,IORDER,ITIME,HT)
C
C *****
C SUBROUTINE BCEQN (PREPARATION FOR BOUNDARY RELATIONS)
C *****
C
C
PARAMETER NW=5,NL=11,L4=4,NV=6
PARAMETER ND=NW*NL,NE=(NW-1)*(NL-1),NB=NW*4
PARAMETER NN=ND*NV,NQ=NN+NB,MC=NQ+1
C
IMPLICIT DOUBLE PRECISION (A-H,O-Z)
COMPLEX B(NB,NN),FST(NN),RAMDA1,RAMDA2,C3,C4,C5,
&      TERM1,TERM2,CG9
COMMON/ZERO/RZG(ND),UZG(ND),VZG(ND),TZG(ND),
&      SZG(ND),PZG(ND),WZG(ND)
COMMON/GIVEN/TC,TS,HO,EO,ES,QL,DEL1,
&      DEL2,BE,GAM,PR,BM
COMMON/FIRST/FST
C
C
ND2=ND*2
ND3=ND*3
ND4=ND*4
ND5=ND*5
C
K1W=NW
K2W=2*NW
K3W=3*NW
K4W=4*NW
RGAM=(GAM-1.0)/GAM
C
HOME=1.0/OME/HT
TS1=1.0/TS
TS2=1.0/TS**2
C1=ES*TS2
ET1=-(ES*TS1+1.0)
C2=(TS-TC)/DEL1
AAT2=SQRT(+0.5+0.5*SQRT(1.0+(4.0*OME*BE*DEL1)**2))
AAT3=0.5*(4.0*OME*BE*DEL1)/AAT2
C
RAMDA1=0.5/DEL1 * DCMPLX(1.0+AAT2,AAT3)
TERM1=(AAT2+1.0+C1*C2*AAT3/(OME*BE))/
&      (2.0*DEL1*DEL2)+C1*QL
TERM2=(AAT3-(AAT2+1.0)*C1*C2/(OME*BE))/(2.0*DEL1*DEL2)+
&      C1*C2/(OME*BE*DEL1*DEL2)
C* TERM1=RAMDA1*(1.0-DCMPLX(0.0,1.0)*C1*C2/OME/BE)/DEL2
C* &      +DCMPLX(0.0,1.0)*C1*C2/OME/BE/DEL1/DEL2
C* &      +C1*QL
C* CG9=0.0
TERMR=TERM1*HT+1.0
TERMI=TERM2*HT

```

```

C
C----- (1) FIRST ORDER B.C.
C
      IF(IORDER.EQ.1) THEN
C
      DO 100 J=1,NW
      I=1+(J-1)*NL
      K= J*NL
      TZI1=1.0/TZG(I)
      TZI2=TZI1/TZG(I)
C
C-----BLOCK TU (TEMPERATURE UPPER CONDITION)
C
      B(K1W+J,I+ND3) -CMPLX(1.00,-HOME)
      B(K1W+J,1+I+ND3)-CMPLX(0.00,+HOME)
      B(K1W+J,I+ND5) -CMPLX(-RGAM,0.0)
C
C-----BLOCK YL (SPECIES LOWER CONDITION)
C
      ENFF→C1*(SZG(K-1)-SZG(K))
      B(K2W+J,K+ND3) -CMPLX(ENFF,0.0)
      B(K2W+J,K+ND4) -CMPLX(HT+1.0,0.0)
      B(K2W+J,K+ND4-1)- CMPLX(-1.0,0.0)
      B(K2W+J, MC) -CMPLX(0.0,0.0)
C
C-----BLOCK VL (Y-VELOCITY LOWER CONDITION)
C
      B(K3W+J,K )-CMPLX(TS**2,0.0)
      B(K3W+J,K+ND2)-CMPLX(1.0,0.0)
      B(K3W+J,K+ND3)-CMPLX(-ES/TS,0.0)
C
C-----BLOCK TL (TEMPERATURE LOWER CONDITION)
C
      B(J,K+ND3) -CMPLX(TERM,TERMI)
      B(J,K+ND3-1)-CMPLX(-1.0,0.0)
      B(J, MC) -CMPLX(0.0,0.0)
C* B(J,K+ND3) -TERM1*HT+1.0
C* B(J,K+ND3-1)-CMPLX(-1.0,0.0)
C* B(J, MC) -HT*CG9
C
      100 CONTINUE
C
C----- (2) SECOND ORDER TIME INDEP. B.C.
C
      ELSEIF(IORDER.NE.1.AND.ETIME.EQ.1) THEN
C
C
      OMBE=1.0/(OME*BE)
      OMB2=0.5*OMBE
      OBDD=0.5/(OME*BE*DEL1*DEL2)
      OME2=2.0*OME
      HOME2=1.0/OME2/HT
      AAT22=SQRT(+0.5+0.5*SQRT(1.0+(4.0*OME2*BE*DEL1)**2))

```



```

      AAT23=0.5*(4.0*OME2*BE*DEL1)/AAT22
C
      TERM21=(AAT22+1.0+C1*C2*AAT23/(OME2*BE))/
&      (2.0*DEL1*DEL2)+C1*QL
      TERM22=(AAT23-(AAT22+1.0)*C1*C2/(OME2*BE))/(2.0*DEL1*DEL2)+
&      C1*C2/(OME2*BE*DEL1*DEL2)
C*      TERM1=1.0/DEL1/DEL2+C1*C2/DEL2+C1*QL
C*      CG9=RAMDA1*C1*FST(JB)*CONJG(FST(JB))*0.5*
C*      &      (1.0-DCMPLX(0.0,1.0)*C1*C2/OME/BE)
C*      &      *(1.0/(DEL1*RAMDA1)-1.0)/DEL2/(DEL1*RAMDA1-1.0)
C*      &      -C2*C3/DEL2-C3*QL
      TERM2R=TERM21*HT+1.0
      TERM2I=TERM22*HT
C
C
      DO 210 J=1,NW
      I=1+(J-1)*NL
      K= J*NL
      TZI1=1.0/TZG(I)
      TZI2=TZI1/TZG(I)
C
C-----BLOCK TU (2ND TEMP. UPPER CONDITION)
C
      B(K1W+J,I+ND3)=CMPLX(1.00,+0.0)
      B(K1W+J,1+I+ND3)=CMPLX(-1.00,+0.0)
      B(K1W+J,MC)=0.0
C
      JA=ND2+K
      JB=ND3+K
      JC=ND4+K
      ENFF=C1*(SZG(K-1)-SZG(K))
C
C-----BLOCK YL (2ND SPECIES LOWER CONDITION)
C
      B(K2W+J,K+ND3)= CMPLX(ENFF,0.0)
      B(K2W+J,K+ND4)= CMPLX(HT+1.0,0.0)
      B(K2W+J,K+ND4-1)= CMPLX(-1.0,0.0)
      B(K2W+J, MC)    --C1*0.5*CONJG(FST(JB))*(FST(JC-1)-FST(JC))
&      +(0.5*C1+1.0/TS)*ENFF*FST(JB)*CONJG(FST(JB))*0.5
C
C-----BLOCK VL (2ND Y-VEL LOWER CONDITION)
C
      PRSL2=1.0
      GAX22=TS1*PRSL2
      B(K3W+J,K)=CMPLX(1.0,0.0)
      B(K3W+J,K+ND3)=CMPLX(TS2,0.0)
      B(K3W+J,MC)=DCMPLX(GAX22,0.0)-0.5*CONJG(FST(K))*FST(JB)*TS1
C
C-----BLOCK TL (2ND TEMP. LOWER CONDITION)
C
      RAMDA2=0.5/DEL1 * DCMPLX(1.0+AAT22,AAT23)
      C3=C1*(0.5*C1-TS1)*FST(JB)*CONJG(FST(JB))*0.5
      CCC2=-C2*(C1*FST(JB))*2*0.5/(OME*BE)**2/DEL1

```

```

      CCC3=(RAMDA2-RAMDA1)*C1*FST(JB)*CONJG(FST(JB))*RAMDA1*0.5
&      *(1.0-CMPLX(0.0,1.0)*C1*C2/(OME*BE))
&      /(DEL1*RAMDA1**2-RAMDA1-CMPLX(0.0,1.0)*OME2*BE)
      TERM2R=HT*(1.0/DEL1/DEL2+C1/DEL2+C1*QL)+1.0
      TERM2I=0.0
C
      B(J,K+ND3)= CMPLX(TERM2R,TERM2I)
      B(J,K+ND3-1)= CMPLX(-1.0,0.0)
      B(J,      MC)= HT*( TC/DEL1/DEL2+C1*FST(JB)*RAMDA1
&      *0.5*CONJG(FST(JB))
&      *(1.0-CMPLX(0.0,1.0)*C1*C2/OME/BE)
&      /(DEL2*(DEL1*RAMDA1-1.0))
&      *(1.0/(DEL1*RAMDA1)-1.0)
&      -C3*C2/DEL2-CMPLX(0.0,1.0)*C2
&      *C1**2*0.5*FST(JB)*CONJG(FST(JB))*2.0*OBDD-C3*QL)
C*      B(J,K+ND3)=TERM1*HT+1.0
C*      B(J,K+ND3-1)= CMPLX(-1.0,0.0)
C*      B(J,MC)      =-HT*CG9
C
210  CONTINUE
C
C----- (3) SECOND ORDER TIME DEP. B.C.
C
      ELSE
C
      OME2=2.0*OME
      HOME2=1.0/OME2/HT
      AAT22=SQRT(+0.5+0.5*SQRT(1.0+(4.0*OME2*BE*DEL1)**2))
      AAT23=0.5*(4.0*OME2*BE*DEL1)/AAT22
C
      DO 220 J=1,NW
      I=1+(J-1)*NL
      K= J*NL
      TZI1=1.0/TZG(I)
      TZI2=TZI1/TZG(I)
C
C-----BLOCK TU (2ND TEMP. UPPER CONDITION)
C
      B(K1W+J,I+ND5)=CMPLX(-RGAM,0.0)
      B(K1W+J,I+ND3)=CMPLX(1.00,+0.0)
      B(K1W+J,1+I+ND3)=CMPLX(-1.00,+0.0)
      B(K1W+J,MC)=-RGAM/GAM*DCMPLX(0.0,1.0)*OME*0.5
C
      JA=ND2+K
      JB=ND3+K
      JC=ND4+K
      ENFF=+C1*(SZG(K-1)-SZG(K))
C
C-----BLOCK YL (2ND SPECIES LOWER CONDITION)
C
      B(K2W+J,K+ND3)= CMPLX(ENFF,0.0)
      B(K2W+J,K+ND4)= CMPLX(HT+1.0,0.0)
      B(K2W+J,K+ND4-1)= CMPLX(-1.0,0.0)

```

```

      B(K2W+J, MC)  = -C1*0.5*FST(JB)*(FST(JC-1)-FST(JC))
&      + (0.5*C1+1.0/TS)*ENFF*FST(JB)*FST(JB)*0.5
C
C-----BLOCK VL (2ND Y-VEL LOWER CONDITION)
C
      B(K3W+J,K) = CMPLX(TS**2,0.0)
      B(K3W+J,K+ND2) = CMPLX(1.0,0.0)
      B(K3W+J,K+ND3) = CMPLX(-ES/TS,0.0)
      B(K3W+J,MC) = C1*(0.5*ES/TS-1.0)*FST(JB)**2*0.5
&      + (FST(JB)-FST(JA))*FST(K)*TS*0.5
C
C-----BLOCK TL (2ND TEMP. LOWER CONDITION)
C
      RAMDA2 = 0.5/DEL1 * DCMPLX(1.0+AAT22,AAT23)
      C3 = C1*(0.5*C1-TS1)*FST(JB)*FST(JB)*0.5
      C4 = -C2*(C1*FST(JB))**2*0.5
&      / (2.0*OME**2*BE**2*DEL1)
      C5 = RAMDA1*(RAMDA2-RAMDA1)*C1*FST(JB)*
&      FST(JB)*(1.0-DCMPLX(0.0,1.0)*C2*C1/OME/BE)*0.5
&      / (DEL1*RAMDA1**2-RAMDA1-DCMPLX(0.0,1.0)*2.0*OME*BE)
C
      TERM1 = RAMDA2*(1.0-DCMPLX(0.0,1.0)*C1*C2
&      *0.5/OME/BE)/DEL2 + DCMPLX(0.0,1.0)
&      *0.5*C1*C2/OME/BE/DEL1/DEL2
&      + C1*QL
      CG9 = C2*C3*0.5/OME/BE/DEL2*(1.0/DEL1-RAMDA2)
&      + (RAMDA2*C4+C5-C4/DEL1)/DEL2-C3*QL
C
      B(J,K+ND3) = TERM1*HT+1.0
      B(J,K+ND3-1) = CMPLX(-1.0,0.0)
      B(J,MC) = HT*CG9
C
220  CONTINUE
C
      ENDIF
C
      RETURN
      END
C

```

```

C      SUBROUTINE DIRICH(A,IORDER)
C      *****
C      SUBROUTINE DIRICH (PREPARATION FOR DIRICHLET CONDITION)
C      *****
C
C      PARAMETER NW=5,NL=11,NV=6
C      PARAMETER ND=NW*NL,NE=(NW-1)*(NL-1),NB=NW*4
C      PARAMETER NN=ND*NV,NQ=NN+NB,MC=NQ+1
C
C      IMPLICIT DOUBLE PRECISION (A-H,O-Z)
C      COMPLEX A(NQ,MC),DIRO,DIR1
C
C      ND2=2*ND
C      ND3=3*ND
C      ND4=4*ND
C      ND5=5*ND
C
C      -----(1) FIRST ORDER DIRICHLET
C      (A) UPPER B.C.
C
C      DIR1= CMPLX(1.0,0.0)
C      DIRO= CMPLX(0.0,0.0)
C
C
C      IF(IORDER.EQ.1) THEN
C      DO 100 J=1,ND,NL
C      K1=J+ND
C      K2=J+ND2
C      K3=J+ND3
C      K4=J+ND4
C      K5=J+ND5
C
C      DO 110 K=1,NQ
C      A(K,MC)=A(K,MC)-A(K,K4)*DIRO
C      A(K,MC)=A(K,MC)-A(K,K5)*DIR1
C      A(K4,K)=DIRO
C      A(K5,K)=DIRO
C      A(K,K4)=DIRO
C      A(K,K5)=DIRO
C      110 CONTINUE
C
C      A(K4,K4)=DIR1
C      A(K5,K5)=DIR1
C      A(K4,MC)=DIRO
C      A(K5,MC)=DIR1
C      100 CONTINUE
C
C      (B) LOWER B.C.
C
C      DO 120 J=NL,ND,NL

```

```

      K1=J+ND
C
      DO 130 K=1,NQ
      A(K,MC)=A(K,MC)-A(K,K1)*DIRO
      A(K1,K)=DIRO
      A(K,K1)=DIRO
130  CONTINUE
      A(K1,K1)=DIR1
      A(K1,MC)=DIRO
120  CONTINUE
C
C----- (2) SECOND ORDER DIRICHLET
C
      ELSE
C      (A) UPPER B.C.
C
      DO 200 J=1,ND,NL
      K1=J+ND
      K2=J+ND2
      K3=J+ND3
      K4=J+ND4
      K5=J+ND5
C
      DO 210 K=1,NN
      A(K,MC)=A(K,MC)-A(K,K4)*DIRO
      A(K4,K)=DIRO
      A(K,K4)=DIRO
C      A(K,K5)=DIRO
210  CONTINUE
C
      A(K4,K4)=DIR1
      A(K4,MC)=DIRO
200  CONTINUE
C
C      (B) LOWER B.C.
C
      DO 220 J=NL,ND,NL
      K1=J+ND
      K5=J+ND5
C
      DO 230 K=1,NN
      A(K,MC)=A(K,MC)-A(K,K1)*DIRO
      A(K1,K)=DIRO
      A(K,K1)=DIRO
230  CONTINUE
      A(K1,K1)=DIR1
      A(K1,MC)=DIRO
220  CONTINUE
C
      ENDIF
C
      RETURN
      END

```

```

C      SUBROUTINE NEUMN(C,IORDER)
C
C      *****
C      SUBROUTINE NEUMN (PREPARATION FOR NEUMANN CONDITION)
C      *****
C
C      PARAMETER NW=5,NL=11,L4=4,NV=6
C      PARAMETER ND=NW*NL,NE=(NW-1)*(NL-1),NB=NW*4
C      PARAMETER NN=ND*NV,NQ=NN+NB,MC=NQ+1
C
C      IMPLICIT DOUBLE PRECISION (A-H,O-Z)
C      COMPLEX C(NN)
C
C      C----- (6) APPLY NEUMANN B.C
C
C      DEFINE BOUNDARY NODES
C      IF(IORDER.EQ.1) THEN
C      DO 100 I=1,NN
C      C(I)=0.0
100  CONTINUE
C      ELSE
C      DO 200 I=1,NN
C      C(I)=0.0
200  CONTINUE
C      ENDIF
C      GRADIENT CALCULATION
C
C      ALL GRADIENTS ARE ASSUMED TO BE ZERO IN THIS CASE
C
C      RETURN
C      END
C

```

```

      SUBROUTINE OUTPUT(IORDER,FST)
C
C *****
C SUBROUTINE OUTPUT( OUTPUT DATA )
C *****
C PRESS : PRESSURE BY DENSITY*TEMPERATURE
C ADMIT : ADMITTANCE(-VELOCITY/PRESSURE)
C RESP : RESPONSE FUNCTION(-BURNING RATE/PRESS)
C
C
C PARAMETER NW=5,NL=11,L4=4,NV=6
C PARAMETER ND=NW*NL,NE=(NW-1)*(NL-1),NB=NW*4
C PARAMETER NN=ND*NV,NQ=NN+NB,MC=NQ+1
C
C
C IMPLICIT DOUBLE PRECISION (A-H,O-Z)
C COMPLEX FST(NN),PRESS(ND),ADMIT(ND)
C
C COMMON/ZERO/RZG(ND),UZG(ND),VZG(ND),TZG(ND),
C & SZG(ND),PZG(ND),WZG(ND)
C COMMON/GIVEN/TC,TS,HO,EO,ES,QL,DEL1,
C & DEL2,BE,GAM,PR,BM
C
C DIMENSION RESP(ND)
C
C ND2=2*ND
C ND3=3*ND
C ND4=4*ND
C ND5=5*ND
C ND6=6*ND
C
C----- (1) SOLUTION OUTPUT
C
C IF(IORDER.EQ.1) THEN
C WRITE(6,900)
C 900 FORMAT(1H1,5X,'FIRST ORDER SOLUTION'//)
C ELSE
C WRITE(6,910)
C 910 FORMAT(1H1,5X,'SECOND ORDER SOLUTION'//)
C ENDIF
C WRITE(6,920)
C 920 FORMAT(5X,'NODE',7X,'DENSITY(RO)',14X,'X-VELOCITY(U)',
C & 13X,'Y-VELOCITY(V)'//)
C DO 100 I=1,ND
C WRITE(6,930) I, FST(I),FST(I+ND),FST(I+ND2)
C 100 CONTINUE
C
C WRITE(6,940)
C 940 FORMAT(5X,'NODE',5X,'TEMPERRATURE(T)',14X,'SPECIES(Y)',
C & 16X,'PRESSURE(P)'//)
C DO 200 I=1,ND
C WRITE(6,930) I, FST(I+ND3),FST(I+ND4),FST(I+ND5)

```

```

200  CONTINUE
C
930  FORMAT(3X,I5,2X,3('(',E10.3,',',E10.3,')',3X))
C
C----- (2) CALCULATE PRESSURE, ADMITTANCE AND RESPONSE FUNCTION
C
      DO 300 I=1,ND
      M=ND2+I
      K=ND3+I
      PRESS(I)=RZG(I)*FST(K)+TZG(I)*FST(I)
      ADMIT(I)=FST(M)/VZG(I)/PRESS(I)
      RESP(I)=DBLE( (RZG(I)*FST(M)+FST(I)*VZG(I))/PRESS(I) )
C      RESP(I)=REAL( (RZG(I)*FST(M)+FST(I)*VZG(I))/PRESS(I) )
300  CONTINUE
C
      IF(IORDER.EQ.1) THEN
      WRITE(6,950)
950  FORMAT(5X,'FIRST ORDER SOLUTION'//)
      ELSE
      WRITE(6,960)
960  FORMAT(5X,'SECOND ORDER SOLUTION'//)
      ENDIF
      WRITE(6,970)
970  FORMAT(5X,'NODE',7X,'PRESSURE(P)',12X,'ADMITTANCE(AD)',
&      12X,'RESPONSE(RE)'//)
      DO 400 I=1,ND
      WRITE(6,980) I, PRESS(I),ADMIT(I),RESP(I)
400  CONTINUE
980  FORMAT(3X,I5,2X,2('(',E10.3,',',E10.3,')',3X),E10.3)
C
      RETURN
      END
C

```



```

SUBROUTINE EIGEN (A)
C
C *****
C SUBROUTINE EIGEN( BOUNDARY CONDITION APPLICATION )
C *****
C VIRTUAL ANF,BNF,CZZ1,
C
C PARAMETER NW=5,NL=11,L4=4,NV=6
C PARAMETER ND=NW*NL,NE=(NW-1)*(NL-1),NB=NW*4
C PARAMETER NN=ND*NV,NQ=NN+NB,MC=NQ+1
C
C IMPLICIT DOUBLE PRECISION (A-H,O-Z)
C COMPLEX EIGAV(NN),EIGBV(NN),EIGV(NN),ANF(NN,NN),BNF(NN,NN),
& CZZ1(NN,NN)
C COMPLEX A(NQ,MC)
C INTEGER ITER(NN)
C
C DO 100 J=1,NN
C DO 100 I=1,NN
C ANF(I,J)=REAL(A(I,J))
C BNF(I,J)=AIMAG(A(I,J))
100 CONTINUE
C
C CALL CONVRT(NN,ANF,NN,BNF,NN,CZZ1,NN)
C CALL SOLVE(NN,ANF,NN,BNF,NN,CZZ1,NN,ITER,EIGAV,EIGBV)
C
C DO 200 IEG=1,NN
C IF(EIGBV(IEG).EQ.0.0) GO TO 200
C EIGV(IEG)=EIGAV(IEG)/EIGBV(IEG)
200 CONTINUE
C
C DO 300 I=1,NN
C WRITE(6,900) EIGV(I)
C WRITE(12,*) EIGV(I)
900 FORMAT(5X, '( ,E10.3,',',E10.3,' )')
300 CONTINUE
C
C STOP
C END
C
C
C

```

```

C
C *****
C SUBROUTINE CONVRT (PREPARATION FOR EIGENVALUE PROBLEM)
C *****
C SUBROUTINE CONVRT
CC
C $( N, A, NA, B, NB, X, NX )
C
C IMPLICIT DOUBLE PRECISION (A-H,O-Z)
C
C COMPLEX A(NA,N)
C COMPLEX B(NB,N)
C COMPLEX W
C COMPLEX X(NX,N)
C COMPLEX Y
C COMPLEX Z
C
C REAL C
C REAL D
C
C INTEGER I
C INTEGER II
C INTEGER IMJ
C INTEGER IM1
C INTEGER IP1
C INTEGER J
C INTEGER JM2
C INTEGER JP1
C INTEGER K
C INTEGER N
C INTEGER NA
C INTEGER NB
C INTEGER NM1
C INTEGER NM2
C INTEGER NX
C
C LUWT = 6
C NM1 = N - 1
C DO 80 I=1,NM1
C D = 0.0
C IP1 = I + 1
C DO 10 K=IP1,N
C Y = B(K,I)
C C = ABS(REAL(Y)) + ABS(AIMAG(Y))
C IF( C.LE.D ) GO TO 9
C D = C
C II = K
9 CONTINUE
10 CONTINUE
C IF( D.EQ.0.0 ) GO TO 78
C Y = B(I,I)
C IF( D.LE.ABS(REAL(Y)) + ABS(AIMAG(Y)) ) GO TO 40

```

```

      DO 20 J=1,N
      Y = A(I,J)
      A(I,J) = A(II,J)
      A(II,J) = Y
20  CONTINUE
      DO 30 J=I,N
      Y = B(I,J)
      B(I,J) = B(II,J)
      B(II,J) = Y
30  CONTINUE
40  CONTINUE
      DO 70 J=IP1,N
      Y = B(J,I)/B(I,I)
      IF( REAL(Y).EQ.0.0 .AND. AIMAG(Y).EQ.0.0 )      GO TO 68
      DO 50 K=1,N
      A(J,K) = A(J,K) - Y*A(I,K)
50  CONTINUE
      DO 60 K=IP1,N
      B(J,K) = B(J,K) - Y*B(I,K)
60  CONTINUE
68  CONTINUE
70  CONTINUE
      B(IP1,I) = CMPLX(0.0,0.0)
78  CONTINUE
80  CONTINUE
C
      DO 100 I=1,N
      DO 90 J=1,N
      X(I,J) = CMPLX(0.0,0.0)
90  CONTINUE
      X(I,I) = CMPLX(1.0,0.0)
100 CONTINUE
C
      NM2 = N - 2
      IF( NM2.LT.1 )      GO TO 270
      DO 260 J=1,NM2
      JM2 = NM1 - J
      JP1 = J + 1
      DO 250 II=1,JM2
      I = N + 1 - II
      IM1 = I - 1
      IMJ = I - J
C
      W = A(I,J)
      Z = A(IM1,J)
      IF( ABS(REAL(W)) + ABS(AIMAG(W)).LE.
$     ABS(REAL(Z)) + ABS(AIMAG(Z)) )      GO TO 140
      DO 120 K=J,N
      Y = A(I,K)
      A(I,K) = A(IM1,K)
      A(IM1,K) = Y
120 CONTINUE
      DO 130 K=IM1,N

```

```

      Y = B(I,K)
      B(I,K) = B(IM1,K)
      B(IM1,K) = Y
130  CONTINUE
140  CONTINUE

```

C

```

      Z = A(I,J)
      IF( REAL(Z).EQ.0.0 .AND. AIMAG(Z).EQ.0.0 )      GO TO 170
      Y = Z/A(IM1,J)
      DO 150 K=JP1,N
      A(I,K) = A(I,K) - Y*A(IM1,K)
150  CONTINUE
      DO 160 K=IM1,N
      B(I,K) = B(I,K) - Y*B(IM1,K)
160  CONTINUE
170  CONTINUE

```

C

```

      W = B(I,IM1)
      Z = B(I,I)
      IF( ABS(REAL(W)) + ABS(AIMAG(W)).LE.
$      ABS(REAL(Z)) + ABS(AIMAG(Z)) )      GO TO 210
      DO 180 K=1,I
      Y = B(K,I)
      B(K,I) = B(K,IM1)
      B(K,IM1) = Y
180  CONTINUE
      DO 190 K=1,N
      Y = A(K,I)
      A(K,I) = A(K,IM1)
      A(K,IM1) = Y
190  CONTINUE
      DO 200 K=IMJ,N
      Y = X(K,I)
      X(K,I) = X(K,IM1)
      X(K,IM1) = Y
200  CONTINUE
210  CONTINUE

```

C

```

      Z = B(I,IM1)
      IF( REAL(Z).EQ.0.0 .AND. AIMAG(Z).EQ.0.0 )      GO TO 249
      Y = Z/B(I,I)
      DO 220 K=1,IM1
      B(K,IM1) = B(K,IM1) - Y*B(K,I)
220  CONTINUE
      B(I,IM1) = CMPLX(0.0,0.0)
      DO 230 K=1,N
      A(K,IM1) = A(K,IM1) - Y*A(K,I)
230  CONTINUE
      DO 240 K=IMJ,N
      X(K,IM1) = X(K,IM1) - Y*X(K,I)
240  CONTINUE
249  CONTINUE

```

C

```
250 CONTINUE
    A(JP1+1,J) = CMPLX(0.0,0.0)
260 CONTINUE
270 CONTINUE
```

C

```
    RETURN
    END
```

SUBROUTINE SOLVE
\$(N, A, NA, B, NB, X, NX, ITER, EIGA, EIGB)

C

IMPLICIT DOUBLE PRECISION (A-H,O-Z)

C

COMPLEX S
COMPLEX W
COMPLEX Y
COMPLEX Z
COMPLEX A(NA,N)
COMPLEX B(NB,N)
COMPLEX X(NX,N)
COMPLEX EIGA(N)
COMPLEX EIGB(N)
COMPLEX ANNM1
COMPLEX ALFM
COMPLEX BETM
COMPLEX D
COMPLEX SL
COMPLEX DEN
COMPLEX NUM
COMPLEX ANM1M1

C

REAL DO
REAL D1
REAL D2
REAL E0
REAL E1
REAL C
REAL EPSA
REAL EPSB
REAL SS
REAL R
REAL ANORM
REAL BNORM
REAL ANI
REAL BNI

C

INTEGER ITER(N)

C

NN = N
ANORM = 0.0
BNORM = 0.0
DO 30 I=1,N
ANI = 0.0
IF(I.EQ.1) GO TO 10
Y = A(I,I-1)
ANI = ANI + ABS(REAL(Y)) + ABS(AIMAG(Y))
10 CONTINUE
BNI = 0.0
DO 20 J=I,N
ANI = ANI + ABS(REAL(A(I,J))) + ABS(AIMAG(A(I,J)))

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      BNI = BNI + ABS(REAL(B(I,J))) + ABS(AIMAG(B(I,J)))
20  CONTINUE
      IF( ANI.GT.ANORM )      ANORM = ANI
      IF( BNI.GT.BNORM )      BNORM = BNI
30  CONTINUE
C
      IF( ANORM.EQ.0.0 )      ANORM = 1.0
      IF( BNORM.EQ.0.0 )      BNORM = 1.0
      EPSA = ANORM
      EPSB = BNORM
40  CONTINUE
      EPSA = EPSA/2.0
      EPSB = EPSB/2.0
      C = ANORM + EPSA
      IF( C.GT.ANORM )      GO TO 40
      IF( N.LE.1 )      GO TO 320
50  CONTINUE
      ITS = 0
      NM1 = NN - 1
60  CONTINUE
      D2 = ABS(REAL(A(NN,NN))) + ABS(AIMAG(A(NN,NN)))
      DO 70 LB=2,NN
      L = NN + 2 - LB
      SS = D2
      Y = A(L-1,L-1)
      D2 = ABS(REAL(Y)) + ABS(AIMAG(Y))
      SS = SS + D2
      Y = A(L,L-1)
      R = SS + ABS(REAL(Y)) + ABS(AIMAG(Y))
      IF( R.EQ.SS )      GO TO 80
70  CONTINUE
      L = 1
80  CONTINUE
      IF( L.EQ.NN )      GO TO 320
      IF( ITS.LT.30 )      GO TO 90
      ITER(NN) = -1
      IF( ABS(REAL(A(NN,NM1))) + ABS(AIMAG(A(NN,NM1))) .GT.
$0.8*ABS(REAL(ANNM1)) + 0.8*ABS(AIMAG(ANNM1)) )      GO TO 999
90  CONTINUE
      IF( ITS.EQ.10 .OR. ITS.EQ.20 )      GO TO 110
C
      ANNM1 = A(NN,NM1)
      ANM1M1 = A(NM1,NM1)
      S = A(NN,NN)*B(NM1,NM1) - ANNM1*B(NM1,NN)
      W = ANNM1*B(NN,NN)*
$      (A(NM1,NN)*B(NM1,NM1) - ANM1M1*B(NM1,NN))
      Y = (ANM1M1*B(NN,NN) - S)/2.0
      Z = CSQRT(Y*Y + W)
      IF( REAL(Z).EQ.0.0 .AND. AIMAG(Z).EQ.0.0 )      GO TO 100
      DO = REAL(Y/Z)
      IF( DO.LT.0.0 )      Z = -Z
100 CONTINUE
      DEN = (Y + Z)*B(NM1,NM1)*B(NN,NN)

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      IF( REAL(DEN).EQ.0.0 .AND.
$      AIMAG(DEN).EQ.0.0 )
$DEN = CMPLX(EPSA,0.0)
      NUM = (Y + Z)*S - W
      GO TO 120
110 CONTINUE
      Y = A(NM1,NN-2)
      NUM = CMPLX(ABS(REAL(ANNM1)) + ABS(AIMAG(ANNM1)),
$          ABS(REAL(Y)) + ABS(AIMAG(Y)) )
      DEN = CMPLX(1.0,0.0)
120 CONTINUE
      IF( NN.EQ.L+1 ) GO TO 140
      D1 = ABS(REAL(A(NN,NN))) + ABS(AIMAG(A(NN,NN)))
      D2 = ABS(REAL(A(NM1,NM1))) + ABS(AIMAG(A(NM1,NM1)))
      E1 = ABS(REAL(ANNM1)) + ABS(AIMAG(ANNM1))
      NL = NN - (L + 1)
      DO 130 MB=1,NL
      M = NN - MB
      EO = E1
      Y = A(M,M-1)
      E1 = ABS(REAL(Y)) + ABS(AIMAG(Y))
      DO = D1
      D1 = D2
      Y = A(M-1,M-1)
      D2 = ABS(REAL(Y)) + ABS(AIMAG(Y))
      Y = A(M,M)*DEN - B(M,M)*NUM
      DO = (DO + D1 + D2)*( ABS(REAL(Y)) + ABS(AIMAG(Y)) )
      EO = EO*E1*( ABS(REAL(DEN)) + ABS(AIMAG(DEN)) ) + DO
      IF( EO.EQ.DO ) GO TO 150
130 CONTINUE
140 CONTINUE
      M = L
150 CONTINUE
      ITS = ITS + 1
      W = A(M,M)*DEN - B(M,M)*NUM
      Z = A(M+1,M)*DEN
      D1 = ABS(REAL(Z)) + ABS(AIMAG(Z))
      D2 = ABS(REAL(W)) + ABS(AIMAG(W))
      LOR1 = 1
      NNORN = N
      DO 310 I=M,NM1
      J = I + 1
      IF( I.EQ.M ) GO TO 170
      W = A(I,I-1)
      Z = A(J,I-1)
      D1 = ABS(REAL(Z)) + ABS(AIMAG(Z))
      D2 = ABS(REAL(W)) + ABS(AIMAG(W))
      IF( D1.EQ.0.0 ) GO TO 60
170 CONTINUE
      IF( D2.GT.D1 ) GO TO 190
      DO 180 K=I,NNORN
      Y = A(I,K)
      A(I,K) = A(J,K)

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&      A(J,K) = Y
      Y = B(I,K)
      B(I,K) = B(J,K)
      B(J,K) = Y
180  CONTINUE
      IF( I.GT.M )      A(I,I-1) = A(J,I-1)
      IF( D2.EQ.0.0 )      GO TO 220
      Y = CMPLX( REAL(W)/D1, AIMAG(W)/D1 )/
      $ CMPLX( REAL(Z)/D1, AIMAG(Z)/D1 )
      GO TO 200
190  CONTINUE
      Y = CMPLX( REAL(Z)/D2, AIMAG(Z)/D2 )/
      $ CMPLX( REAL(W)/D2, AIMAG(W)/D2 )
200  CONTINUE
      DO 210 K=I,NNORN
      A(J,K) = A(J,K) - Y*A(I,K)
      B(J,K) = B(J,K) - Y*B(I,K)
210  CONTINUE
220  CONTINUE
      IF( I.GT.M )      A(J,I-1) = CMPLX(0.0,0.0)
      Z = B(J,I)
      W = B(J,J)
      D1 = ABS(REAL(Z)) + ABS(AIMAG(Z))
      D2 = ABS(REAL(W)) + ABS(AIMAG(W))
      IF( D1.EQ.0.0 )      GO TO 60
      IF( D2.GT.D1 )      GO TO 270
      DO 230 K=LOR1,J
      Y = A(K,J)
      A(K,J) = A(K,I)
      A(K,I) = Y
      Y = B(K,J)
      B(K,J) = B(K,I)
      B(K,I) = Y
230  CONTINUE
      IF( I.EQ.NM1 )      GO TO 240
      Y = A(J+1,J)
      A(J+1,J) = A(J+1,I)
      A(J+1,I) = Y
240  CONTINUE
      DO 250 K=1,N
      Y = X(K,J)
      X(K,J) = X(K,I)
      X(K,I) = Y
250  CONTINUE
      B(J,I) = CMPLX(0.0,0.0)
      IF( D2.EQ.0.0 )      GO TO 310
      Z = CMPLX( REAL(W)/D1, AIMAG(W)/D1 )/
      $ CMPLX( REAL(Z)/D1, AIMAG(Z)/D1 )
      GO TO 280
270  CONTINUE
      Z = CMPLX( REAL(Z)/D2, AIMAG(Z)/D2 )/
      $ CMPLX( REAL(W)/D2, AIMAG(W)/D2 )
280  CONTINUE

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DO 290 K=LOR1,J
A(K,I) = A(K,I) - Z*A(K,J)
B(K,I) = B(K,I) - Z*B(K,J)
290 CONTINUE
B(J,I) = CMPLX(0.0,0.0)
IF( I.LT.NM1 ) A(I+2,I) = A(I+2,I) - Z*A(I+2,J)
DO 300 K=1,N
X(K,I) = X(K,I) - Z*X(K,J)
300 CONTINUE
310 CONTINUE
GO TO 60
320 CONTINUE
EIGA(NN) = A(NN,NN)
EIGB(NN) = B(NN,NN)
IF( NN.EQ.1 ) GO TO 330
ITER(NN) = ITS
NN = NM1
IF( NN.GT.1 ) GO TO 50
ITER(1) = 0
GO TO 320
330 CONTINUE
M = N
340 CONTINUE
ALFM = A(M,M)
BETM = B(M,M)
B(M,M) = CMPLX(1.0,0.0)
L = M - 1
IF( L.EQ.0 ) GO TO 370
350 CONTINUE
L1 = L + 1
SL = CMPLX(0.0,0.0)
DO 360 J=L1,M
SL = SL + B(J,M)*(BETM*A(L,J) - ALFM*B(L,J))
360 CONTINUE
Y = BETM*A(L,L) - ALFM*B(L,L)
IF( REAL(Y).EQ.0.0 .AND.
$ AIMAG(Y).EQ.0.0 )
$Y = CMPLX( (EPSA+EPSB)/2.0, 0.0 )
B(L,M) = -SL/Y
L = L - 1
370 CONTINUE
IF( L.GT.0 ) GO TO 350
M = M - 1
IF( M.GT.0 ) GO TO 340
M = N
380 CONTINUE
DO 400 I=1,N
S = CMPLX(0.0,0.0)
DO 390 J=1,M
S = S + X(I,J)*B(J,M)
390 CONTINUE
X(I,M) = S
400 CONTINUE

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      M = M - 1
      IF( M.GT.0 )      GO TO 380
      M = N
410  CONTINUE
      SS = 0.0
      DO 420 I=1,N
      R = ABS(REAL(X(I,M))) + ABS(AIMAG(X(I,M)))
      IF( R.LT.SS )      GO TO 418
      SS = R
      D = X(I,M)
418  CONTINUE
420  CONTINUE
      IF( SS.EQ.0.0 )      GO TO 440
      DO 430 I=1,N
      X(I,M) = X(I,M)/D
430  CONTINUE
440  CONTINUE
      M = M - 1
      IF( M.GT.0 )      GO TO 410
999  CONTINUE
      RETURN
      END

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